Uncertainty and Market Efficiency: An Information Choice Perspective *

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Abstract

We develop an information choice model where information costs are sticky and co-move with firm-level intrinsic uncertainty as opposed to temporal variations in uncertainty. Incorporating analysts' forecasts, we predict a negative relationship between information costs and information acquisition, as proxied by the predictability of analysts' forecast biases. Finally, the model shows a contrasting pattern between information acquisition and intrinsic and temporal uncertainty, where intrinsic uncertainty strengthens return predictability of analysts' biases through the information cost channel, while temporal uncertainty weakens it through the information benefit channel. We empirically confirm these opposing relationships that existing theories struggle to explain.

Keywords: information choice, sticky information cost, volatility, analysts' forecasts, machine learning, market expectations

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1 Introduction

Is the financial market more or less efficient when uncertainty level is high? In this paper, we explore information-choice based models to provide an alternative perspective.[1] In these models, investors have limited information processing capacity (Sims, 2003), and market efficiency varies with investors' information acquisition decisions — more information acquisition, higher market efficiency.

Relevant literature has not directly addressed this question. Indeed, existing theories have ambiguous predictions on whether higher uncertainty is associated with more or less information acquisition (Van Nieuwerburgh and Veldkamp, 2010), and extant empirical evidence is mixed.[2]

We aim to make theoretical and empirical contributions along this line of inquiry. Theoretically, we propose an information-choice model with three key novel features. The first is that different firms impose persistently different information processing costs on investors. Intuitively, a young pharmaceutical firm should require more information processing costs from investors, as opposed to an established retailer. Yet, an approach typically taken in the existing literature is to assume homogeneous costs across firms (Biao, maybe some literature as you mentioned to me).

The second, and the key feature of the model, is that firm-level information processing costs are related to uncertainty. Specifically, these costs mainly vary with firm-level intrinsic uncertainty, rather than with temporal changes in uncertainty. We term this feature the "Sticky Information Costs" (SIC) hypothesis, which has been largely overlooked in the literature, despite its intuitive appeal and importance in driving investors' information acquisition decisions facing uncertainty.[3]

To demonstrate the the SIC hypothesis, we consider two specific firms: Regeneron Pharmaceuticals Inc. (NASDAQ:REGN), a pharmaceutical company known for its cutting-edge innovations in biotech and pharmaceuticals, and Walmart Inc. (NYSE:WMT), an American brick & mortar retail chain with a long-running and straightforward business

[1] In models featuring investors with full-information rational expectation (FIRE), the market stays fully efficient despite objective uncertainty variations, both across firms and over time.

[2] Existing empirical studies find that investors appear to pay more attention to information when uncertainty is high, supporting the benefit channel (Bonsall et al., 2020; Benamar et al., 2021). Fuster et al. (2022); Conlon et al. (2018) find evidence in survey and experimental setting that individuals with higher prior uncertainty do not update more when receiving new information, suggesting the important role of the cost channel.

[3] Indeed, the property of information costs is an understudied and growing area in information choice (Blankespoor et al., 2019), as previous models have primarily focused on the relationship between uncertainty and the benefits of acquiring information.

model. Analyzing the future profitability of Regeneron requires deep expertise to assess the new drugs it develops than projecting Walmart's future revenue. Furthermore, Regeneron has a shorter history of public data than Walmart, so overall less data is available to acquire.[4] The SIC simply states that this difference in processing costs should be mainly driven by the difference in intrinsic uncertainty and less so by temporal variation in uncertainty.

Figure 1: An Illustrative Example of the "Sticky Information Cost"

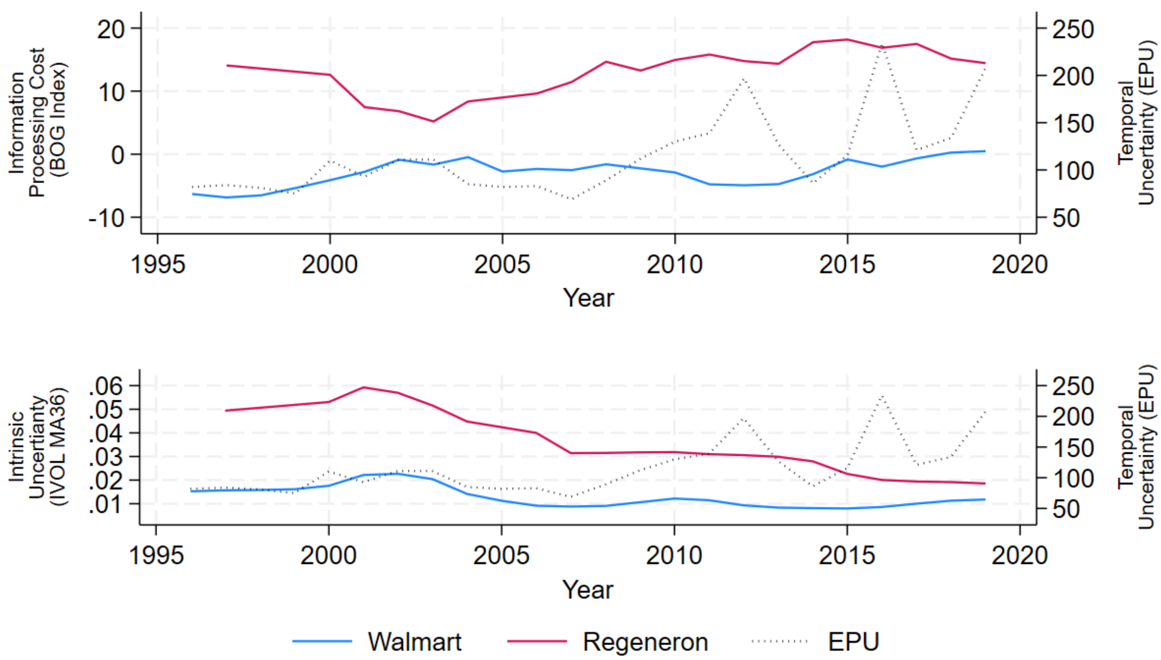


Figure 1 plots a measure of information costs of these two companies along with proxies of the two types of uncertainty. The upper panel shows the information processing costs, i.e. the readability of their annual reports (the Bog Index as proposed in Bonsall et al. (2017)) over time along with the EPU, an index of aggregate economic uncertainty proposed by XXX. The figure shows Regeneron has persistently higher information processing costs than Warmart over the last 25 years, and their differences are not much impacted by the variation of EPU. In contrast, as the lower panel shows, these firms' intrinsic uncertainty — as proxied by their 36-month moving averages of idiosyncratic volatility – follow a similar

[4]Regeneron went public in April 1991, while Walmart stock started trading in August 1972.

pattern as in the upper panel, indicating strong relation. These patterns capture the essence of the SIC hypothesis.

The third feature of the model is the presence of sell-side analysts, a large information intermediary important to the efficiency of the market (Kothari et al., 2016). The analysts in our model produce ex-ante biased forecasts ("EHB") as in reality. Furthermore, investors have free access to these forecasts, which, if not de-biased by investors, will lead to return predictability. On the contrary, if investors fully de-bias these biases, there should be no return predictability.

Building on these features, we derive the equilibrium predictions of our model with respect to the relationship between information processing costs, uncertainty variation and return predictability of EHB. Investors in our model face a trade-off when acquiring information with their limited information processing capacity to de-bias analysts' forecasts. Intuitively, facing uncertainty, investors have more incentive to acquire information, as every bit of information is valuable (the "benefit channel"). However, acquiring the additional information does not necessarily cost the same. With higher uncertainty, every bit of information may be more costly (the "cost channel"). The two channels deliver the opposite relationships between uncertainty and return predictability: if uncertainty arises mostly through the benefit channel while the cost does not increase as much, higher uncertainty leads to greater information acquisition in equilibrium and therefore less return predictability of EHB, and vice versa.

In sum, the model yields three new implications related to information costs, information acquisition, and uncertainty. First, there is a stronger relationship between information costs and intrinsic uncertainty than information costs and temporal uncertainty. Second, there is a negative relationship between information costs and information acquisition. Finally, variations in temporal uncertainty and intrinsic uncertainty should lead to contrasting patterns information acquisition, which in turn leads to contrasting patterns in the return predictability of EHB.

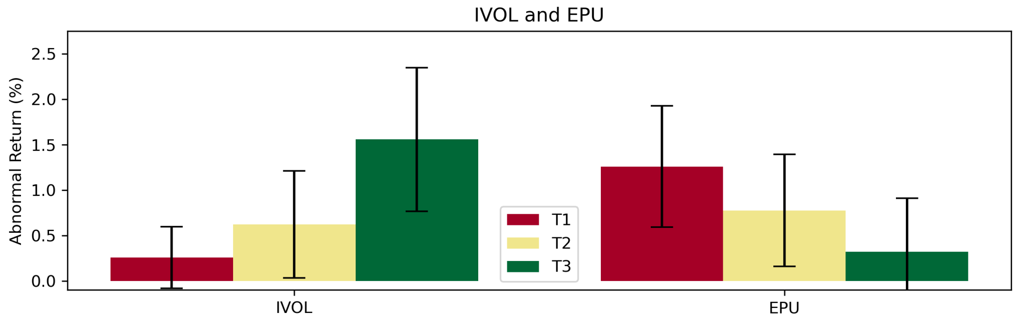
To test the first prediction, we use direct measures of information processing costs proposed in the literature and construct ML-based earnings ex-ante analyst biases similar to that of van Binsbergen et al. (2022). Validating the SIC, we find that the information cost measures are persistent over time, with large autoregressive coefficients. Furthermore, they exhibit more positive correlations with measures of intrinsic uncertainty relative to measures of temporal uncertainty.

Second, we evaluate the relationship between information processing costs and information acquisition. The model predicts that as information costs increase, the amount of information acquired by investors decreases. We use the return predictability of EHB as a measure of information acquisition. In a situation where investors fully de-bias analysts forecasts, there should be no return predictability generated from the predictable component of said forecasts. Conversely, our model predicts that for situations where the information cost channel dominates, investors will acquire less information to de-bias the analysts forecasts, leading to stronger return predictability of EHB. To test this, we sort firms into terciles based on a measure of information cost. Within each tercile, we we form quintile portfolios based on EHB. We find that firms with higher information costs generate abnormal returns of a larger magnitude of a long-short EHB portfolio relative to firms with lower information costs.

Using our findings above, we then can evaluate whether a contrasting relationship exists between investors' information acquisition and variations in intrinsic and temporal uncertainty. We expect the information cost channel to play a dominant role for variations in intrinsic uncertainty as the costs to acquire information varies across firms much more than it does across time. As such, we expect abnormal returns to be greater firms with high intrinsic uncertainty relative to low intrinsic uncertainty. Conversely, as information costs are sticky across time, the information benefit channel is expected to dominate in the time-series, leading to lower abnormal returns in periods of high temporal uncertainty relative to low temporal uncertainty.

In the same manner as the tests for information cost and information acquisition, we first sort our sample into terciles based on measures of temporal and intrinsic uncertainty. Within each tercile, we we form quintile portfolios based on EHB. Our results support the model's predictions: the magnitude of a long-short EHB abnormal returns varies with uncertainty in opposite ways when considering variations in intrinsic vs. temporal uncertainty, as illustrated in Figure 2. In this figure, the left half of the panel shows that the (Fama-French Five-Factor [FF5]) alphas of the long-short portfolios sorted on EHB are the *highest* in the high intrinsic uncertainty tercile measured by firm-level idiosyncratic volatility ("IVOL"), while the right panel shows that the alphas are the *highest* in the low temporal uncertainty tercile measured by EPU.

Figure 2: The Opposite Relationships Between Uncertainty and Return Predictability



Note: uncertainty levels are the lowest in tercile 1 (T1) and highest in tercile 3 (T3). The whiskers indicate the 95% confidence interval around point estimates.

We show that this contrasting pattern is robust across multiple measures of intrinsic and temporal uncertainty. These results strongly support the our model and suggest that the information cost channel dominates for variations in intrinsic uncertainty and the information benefit channel dominates for variations in temporal uncertainty.

We investigate a broader set of variations in anomaly return predictability are consistent with the contrast pattern. First, we document a strong size effect in the return predictability of EHB, which is consistent with a smaller information benefit and higher information cost among small-cap firms. Second, we analyze two prominent earnings-related return anomalies related to analysts' revisions and announcement day returns. Using these two anomalies to proxy for investors' inefficient processing of earnings-related information, we similarly find an overall positive relation between temporal uncertainty and information acquisition but a negative relation between intrinsic uncertainty and information acquisition.

Finally, we explore whether alternative explanations based on information demand, behavioral biases, or limits of arbitrage can explain the contrasting cross-sectional versus time-series relationships between uncertainty and the degree to which investors efficiently process analysts' forecasts. Using EDGAR downloads from Ryans (2017) as a proxy for information demand, the magnitude of EHB as a proxy for behavioral biases, and the effective bid-ask spreads as a proxy for the trading friction, we find that these alternative stories struggle to explain our empirical findings. Thus, the relation between uncertainty and anomaly returns offers a valuable empirical moment that helps distinguish between information choice theory and these competing theories.

1.1 Related Literature

Our paper contributes to the literature on the relation between uncertainty and investors' information acquisition. A number of prior studies (e.g., Van Nieuwerburgh and Veldkamp, 2010; Benamar et al., 2021; Dávila and Parlatore, 2023; Andrei et al., 2023) build theoretical models that analyze the relationship between uncertainty and information acquisition.[5] We add to the theoretical analysis of the relationship between uncertainty and investors' information acquisition through two innovations. First, conventional literature such as Kacperczyk et al. (2016) and Van Nieuwerburgh and Veldkamp (2010) usually focuses on the benefit channel of information acquisition, letting the information acquisition cost be homogeneous across stocks. Here investors' learning decision depends on the benefit only. Prevailing empirical evidence generally finds a positive relation between uncertainty and investors' information acquisition (Loh and Stulz, 2018; Benamar et al., 2021; Andrei et al., 2023), supporting the information benefit channel that higher uncertainty amplifies the marginal benefit of information.

We incorporate heterogeneous learning cost in the cross-section of stocks and generate new insights on the impact of information cost on optimal information cost and uncertainty. Through this new channel, we find that the relation between uncertainty and information acquisition could have opposite direction in the time-series vs. in the cross-section. Our findings demonstrate that the costs of information acquisition also play a vital role in driving the relationship between uncertainty and investors' information acquisition.

While several recent studies have highlighted the role of information acquisition costs in information acquisition (e.g., Blankespoor et al., 2019; Chen et al., 2022; Fuster et al., 2022; Huang et al., 2022), our study is the first to propose and test the hypothesis that information cost can have distinct relationships with cross-sectional and time-series variations in uncertainty. Our results highlight the necessity to distinguish between these two types of variations in uncertainty to accurately model the information cost channel, which holds significant implications for future research. Conversely, as both types of uncertainty variations positively influence the information benefit, such a distinction may not be necessary when modeling the information benefit channel.

[5] Dávila and Parlatore (2023) develop a general equilibrium model that elucidates the intricate relationship between uncertainty and price informativeness. As defined in Dávila and Parlatore (2023) and Dávila and Parlatore (2018), price informativeness is determined by the regression of prices on future payoffs, assessing the extent to which prices predict future asset payoffs. This concept of price informativeness differs from our approach to measuring information acquisition, which involves running regressions of future returns on ex-ante biases in analysts' forecasts to determine whether investors completely adjust for this bias.

Second, our model is the among first to take analyst forecast as a information intermediary in the investors learning process. The model explicitly derives how analyst forecast bias is incorporated into prices and predicts returns. The model illustrates the relation between information acquisition (i.e., de-biasing) and return predictability of analyst forecast bias documented in the literature (van Binsbergen et al., 2022). As such, our paper also relates to the literature that aims to understand the role of analysts' forecasts in shaping the markets' earnings expectations, as summarized in Kothari et al. (2016). Specifically, the paper provides an information choice perspective to explain the long-standing puzzle of why investors do not fully unravel analysts' bias (Frankel and Lee, 1998; So, 2013; van Binsbergen et al., 2022). Furthermore, we present evidence that shows that the proposed model can potentially explain a broader range of earnings-related return-predictability patterns.

2 An Information Choice Model with Two Types of Uncertainty Variation

2.1 Theoretical Background: A Information Choice Model with Analyst Forecast as Information Intermediary

We present our theoretical framework below. The model has three periods $t = 0, 1, 2$. There are n risky assets and one riskless asset in the market. The risky assets include $n-1$ stocks and one composite asset (the market portfolio). At $t = 0$, analysts produces forecasts of the risky assets and a continuum of investors with a measure of one choose to allocate their attention across different assets. Investors can allocate attention to *de-bias* analysts' forecast (or, in other words, get a private signal based on the forecast). At $t = 1$, the investor chooses a portfolio of assets based on their posterior. At $t = 2$, asset payoffs are realized.

2.1.1 Setup

Assets The model has one riskless and n risky assets. The riskless asset is normalized to have unit return and infinity supply. Risky assets (stocks) have net positive supplies, and random payoffs f_i at $t = 2$ with the following factor structure:

$$\begin{aligned} f_i &= \mu_i + \beta_i z_n + z_i, \quad \forall i = 1, \cdots, n-1 \\ f_n &= \mu_n + z_n \end{aligned}$$

where μ_i is the expected payoff of asset i, and $z_i \sim N(0, \sigma_i)$ is an asset i-specific payoff shock (risk factor) that are independent from each other, and σ_i is the variance of the shock. z_n can be interpreted as an aggregate shock to all stocks (i.e., the market factor). We denote the covariance matrix of z as a diagonal Σ with the entry (i, i) being σ_i.

We argue that the firm-specific uncertainty contains two parts. One is determined by the firm's intrinsic uncertainty, e.g. complexity in business model. This part is sticky and varies across firms. The other part is due to temporary fluctuations e.g. resolution of uncertainty or changes in the aggregate information environment. As such, we propose the following structure of the firm-specific shock variance.

$$\sigma_i = \sigma_i^F + \sigma_i^S, \quad \forall i = 1, \cdots, n-1 \tag{1}$$

where $\sigma_i^S \equiv \phi_i \sigma^S$ represents the *temporal uncertainty* component in σ_i. This is motivated by the finding from Herskovic et al. (2016) that firms' idiosyncratic volatility share a common factor, i.e., CIV, even though the residuals–z_i–are uncorrelated. This common factor varies across time and firms load on it heterogeneously. The fixed loading ϕ captures firm's riskiness as an exposure to the temporal volatility. In this model, we consider this parameter as exogenous and unrelated to the information acquisition decisions. In addition to the component driven by the temporal uncertainty, we add a further layer of *intrinsic uncertainty* σ_i^F that is related to the underlying business of the firm. This can be represented as the persistent cross-sectional difference of the idiosyncratic volatility after accounting for the exposure to the CIV factor.

Following Kacperczyk et al. (2016), we focus on factors where the payoff is $\tilde{f} = \Gamma^{-1}\mu + z$, where Γ is a n by n matrix that maps the risk factor z to the asset mean-zero payoffs $f - \mu$.[6] The payoff of a risk factor is payoff to a portfolio of the underlying asset. The advantage of dealing with risk factors is the analytical tractability given the independence among risk factors.

For simplicity, we assume a *fixed supply* of each factor, denoted as x_i. The literature usually imposes a supply noise, which prevents the price from fully revealing the fundamental value. In our framework, however, the aggregated private signal cannot fully reveal the fundamental z, since all signal are based on a common analyst forecast. To better clarify our mechanism, we neglect the supply noise. In Appendix C.2, we present a generalized version of the model with skilled and unskilled investors and supply noise. The results are consistent

[6]Specifically, Γ's diagonal element is 1 and the last column is the vector given by $\{\beta_i\}_{i=1}^{n-1}$. All other entries are zero.

with the simplified model.

Preference Let W_0 and W_j be the initial ($t = 0$) and the final ($t = 2$) wealth for investor j. We assume investors have mean-variance utility over final wealth. We use E_j (V_j) to denote investor j's expectation (variance) conditional on their information at time $t = 1$.

At $t = 1$, investors choose the holding of assets q_j to maximize the expected utility

$$U_{1j} = E_j [W_j] - \frac{\gamma}{2} \mathrm{V}_j [W_j] \tag{2}$$

subjective to the budget constraint $W_j = W_0 + q_j'(f - p)$. Here γ is the risk aversion coefficient. q_j and p are n by 1 vectors of asset holdings and prices, respectively. Using $\tilde{p} = \Gamma^{-1} p$ and $\tilde{q}_j = \Gamma' q_j'$, we can write the problem to be maximizing utility by choosing the holdings of factors with the constraint $W_j = W_0 + \tilde{q}_j'(\tilde{f} - \tilde{p})$

Prices In equilibrium, the price of factors and assets can be determined by the market clearing condition:

$$\int \tilde{q}_{ij} dj = x_i \quad \forall i = 1, \cdots, n \tag{3}$$

The left-hand side is the aggregate demand for risk factors and the right-hand side is the aggregate supply.

De-biasing and signal structure For each factor, analysts produces a consensus forecast of the payoff at time $t = 0$. $AF_i = z_i + B_i$, where $B_i \sim N(0, \sigma_i^B)$ is the bias of the forecast, with σ_i^B being the bias variance.

We assume that the variance of the analyst forecast bias is proportional to the prior variance, i.e., $\sigma_i^B = \rho \sigma_i$. This assumption captures the intuition that analysts produce noisier forecast in times of heightened uncertainty and for firms that are innately more uncertain, consistent with Loh and Stulz (2018). In addition, Loh and Stulz (2018) find that investors rely more on analyst forecasts in times with higher prior uncertainty, which implies that analyst forecast bias precision decreases more slowly than the prior precision in bad times. As such, we further assume $\rho \leq 1$.

For simplicity, z_i and B_i are assumed to be independent. Investors are aware of the bias and must allocate attention to acquire information in order to de-bias the forecast. Specifically at $t = 0$, investor j can exert effort to de-bias a fraction $b \in [0, 1]$ of B_i. After de-biasing, the investor obtains a more precise signal on the fundamental. For factor i, investor j's signal is

$$\eta_{ij} = z_i + \varepsilon_{ij} = z_i + (1 - b_{ij})B_i$$

The vector of signal noise for investor j is distributed as $N(0, \Sigma_{\eta j})$ where $\Sigma_{\eta j}$ is a diagonal matrix with the ith diagonal element given by $\sigma_{\eta,ij}$. We denote the relative precision of the processed signal compared to the original signal as θ_{ij}, given by $\theta_{ij} = \frac{\tau_{ij}^{\eta}}{\tau_i^B} = \frac{1}{(1-b_{ij})^2} > 1$. This measure quantifies the extent to which signal precision improves through the de-biasing process. In the next subsection, we discuss how to specify our information cost function based on the improvement in the precision.

Information cost De-biasing analyst forecasts requires attention and effort, which we formalize through the concept of information acquisition cost. The literature has proposed various functional forms for information costs, including entropy-based costs (Van Nieuwerburgh and Veldkamp, 2010) and additive costs (Van Nieuwerburgh and Veldkamp, 2010; Kacperczyk et al., 2016). In this paper, we follow Avramov et al. (2022) and adopt a quadratic information cost function. This specification ensures the existence of a well-defined, interior solution for optimal information acquisition while also capturing the intuitive economic principle that the marginal cost of improving forecast precision increases as information acquisition intensifies. Specifically, for an investor j to de-bias the analyst forecasts for factor i, the cost is a quadratic function on the de-biasing effort,

$$c_{ij}(\theta_{ij}) = \frac{\kappa_i}{2}(\theta_{ij} - 1)^2 = \frac{\kappa_i}{2}\left(\frac{\tau_{ij}^{\eta}}{\tau_i^B} - 1\right)^2 \tag{4}$$

where κ_i determines the marginal cost of increasing signal precision. This cost function satisfies several properties: it is non-negative, increasing, and convex in de-biasing level. In addition, the cost is zero when there is no information acquisition and goes to infinity when $b \to 1$, which indicates that investor can never fully de-bias the analyst forecast to obtain the exact value of the fundamental.

Assumption 1 (**Sticky Information Cost**). *A firm' information cost is proportional to the its intrinsic uncertainty,* $\kappa_i = \psi\sigma_i^F$.

This is the key assumption and main innovation in our model. It claims that investors' marginal costs to process information are slow moving and vary with the firm's fundamental characteristics, such as technology and business models. This is motivated by the intuition that the information processing costs are persistent, as shown in Figure 1.

Sticky Information Cost (SIC) implies a distinctive relationships between investors' information processing costs based variations in temporal and intrinsic uncertainty. Across firms, those with more advanced technology and more complicated business models require investors to incur higher marginal costs to de-bias analysts forecasts. Anecdotally, research analysts working for buy-side firms specializing in biotech companies typically have high entry costs such as requiring experience in R&D or holding advanced medical degrees while the entry requirement for analysts in retail sectors is relatively lower. At the same time, these more complicated firms are also those with higher ex-ante intrinsic volatility; for example, Regeneron vs. Walmart, as shown in Figure 1. As a result, SIC implies a positive relationship between information processing costs and intrinsic uncertainty.

At the same time, SIC assumes that the processing costs do not correlate as strongly with temporary variations in uncertainty over time. Intuitively, if a young technology firm's volatility spikes due to macroeconomic conditions or earnings, the SIC assumption posits that the costs for analyzing this firm should not significantly increase as the firms' fundamental characteristics have not noticeably changed.

In section 4, our empirical evidence shows that it is indeed the case: a measure of information cost is positively correlated with measures of intrinsic uncertainty while uncorrelated with measures of temporal uncertainty (see Figure 5).

Posteriors Based on the private signal, an investor updates her beliefs about the factors by forming a Bayesian posterior with mean and variance.[7]

$$\hat{\mu}_j \equiv E_j\left[z|\eta_j\right] = \hat{\Sigma}_j \Sigma_{\eta j}^{-1} \eta_j, \quad \hat{\Sigma}_j^{-1} = \Sigma^{-1} + \Sigma_{\eta j}^{-1}$$

where $\hat{\Sigma}_j$ is investor j's posterior variance on the factors z. From a time $t=0$ perspective, $\hat{\mu}$ is normally distributed with zero mean and variance-covariance matrix $V_0\left[\hat{\mu}_j\right] = \Sigma - \hat{\Sigma}_j$ according to the law of total variance.

Given the homogeneity of skilled investors, we study a symmetric equilibrium where every investor will choose the same level of de-biasing for a given factor and obtain the same posterior precision Σ_η^{-1}. In the symmetric equilibrium, the aggregate posterior precision is $\bar{\Sigma}^{-1} = \int \hat{\Sigma}_j^{-1} dj = \Sigma^{-1} + \Sigma_\eta^{-1}$,

[7] For simplicity, we do not include noisy traders and uninformed investors in our benchmark model. As shown in Lemma 1, the price fully reveals the private signal. Therefore, price signal is not used in investors' information updating. In Appendix C.2 we present the fully-specified model with heterogeneous investors and supply noise following the standard literature. The results hold consistently.

Equilibrium The equilibrium of the model is determined by the following optimization problem. An investor maximizes time $t = 0$ expected utility by choosing the signal precision (or de-biasing)

$$U_{0j} \equiv E_0 \left(E_j \left[W_j \right] - \frac{\gamma}{2} \mathrm{V}_j \left[W_j \right] \right) - \sum_{i=1}^{n} c_{ij} \tag{5}$$

2.1.2 Solutions

We solve the model backward. First, we solve the portfolio optimization problem at $t = 1$, taking the information acquisition and posterior beliefs as given. In this step, we can also derive the equilibrium price. Second, we derive the optimal information acquisition problem and produce propositions about the relation between uncertainty and information acquisition.

Portfolio allocation The optimization problem is given by

$$\begin{array}{ll} \max\limits_{\tilde{q}_j} & U_{1j} = E_j \left[W_j \right] - \frac{\gamma}{2} \mathrm{V}_j \left[W_j \right] \\ \text{s.t.} & W_j = W_0 + \tilde{q}_j{}'(\tilde{f} - \tilde{p}) \end{array}$$

which gives the solution

$$\tilde{q}_j = \frac{1}{\gamma} \hat{\Sigma}_j^{-1} \left[E_j(\tilde{f}) - \tilde{p} \right] \tag{6}$$

Then we plugin this demand function to the market clear condition, $\int \tilde{q}_j dj = \overline{x} + x$, and obtain the following Lemma.

Lemma 1. *The equilibrium price of the factors is*

$$\tilde{p} = A_0 + A_z z + A_B B$$

where

$$\begin{aligned} A_0 &= \Gamma^{-1} \mu - \gamma \bar{\Sigma} x \\ A_z &= \bar{\Sigma} \Sigma_\eta^{-1} \\ A_B &= \bar{\Sigma} \Sigma_\eta^{-1} (\mathbf{I} - \mathbf{b}) \end{aligned}$$

$\bar{\Sigma}$ *and* Σ_η *are given below in the proof.*

Proof. See Appendix C $\square$

Lemma 1 shows that the equilibrium price is a linear function on the fundamental shocks z and analysts' bias B. The price loading on analysts' bias is proportional to that on the fundamental shock, with the proportion being $1 - b_i$, i.e., the investors' de-biasing level of factor i in equilibrium. When investors fully de-bias B_i ($b_i = 1$), the price is not related to the bias. In contrast, if investors do not de-bias B_i at all ($b_i = 0$), the price respond to the bias as much as it would to the fundamental shock.

Lemma 1 also tells us the excess returns of each stock, defined by $r^e = f - \Gamma\tilde{p}$. Specifically, we derive the following corollary.

Corollary 1. *The excess return of stock i is*

$$
\begin{aligned}
r_i^e &= \gamma\bar{\sigma}_i x_i + \beta_i r_n^e + \zeta_i^z z_i - \zeta_i^B B_i, \quad \forall i = 1, \cdots, n-1 \\
r_n^e &= \gamma\bar{\sigma}_n x_n + \zeta_n^z z_n - \zeta_n^B B_n
\end{aligned}
$$

where

$$
\zeta_i^z = \tfrac{\bar{\sigma}_i}{\sigma_i}
$$
$$
\zeta_i^B = \tfrac{\bar{\sigma}_i}{\sigma_{\eta i}}(1 - b_i)
$$

Proof. See Appendix C □

Per Corollary 1, the excess return of a stock depends on four parts: (i) a constant determined by its idiosyncratic volatility and supply; (ii) a part that depends on the market excess return and its exposure (CAPM); and (iii) fundamental shocks z_i, and (iv) analyst forecast bias B_i with stock-specific loadings.

Corollary 2. *The analysts' bias B_i negatively predict stock excess return. In addition, the predictability is weaker when the de-biasing activity b_i is stronger.*

Proof. See Appendix C □

Corollary 2 shows that analysts' bias predicts returns negatively ($\zeta_i^B < 0$), consistent with the empirical findings. In addition, de-biasing affects return predictability of analyst forecast bias in two ways: first, more de-biasing leads to higher signal precision relative to the prior precision, and thus returns depends more on signals, as reflected in the term $\frac{\bar{\sigma}_i}{\sigma_{\eta i}} = \frac{\tau_{\eta i}}{\bar{\tau}_i}$. In this case, the bias contained in the signal predicts return stronger. Second, more de-biasing decrease the fraction of bias that is incorporated into the return, lowering the return predictability, which is captured by the term $1 - b_i$.[8] Corollary 2 says that under our model assumption, the impact of de-biasing always dominates. This is because that the analysts' forecast is already sufficiently precise, given that $\sigma_i^B < \sigma_i$. Consequently, the marginal effect of increasing precision through de-biasing is outweighed by the marginal effect of the de-biasing process itself, as demonstrated in the proof. The impact of de-biasing on return predictability is the main mechanism in our model.

Information decision At $t = 0$, investors choose posterior precision of the the de-biased through information acquisition to maximize time $t = 0$ expected utility U_{0j}.

The proof of Lemma 2 shows that the time $t = 0$ utility can be written as the following form

$$U_{0j} = constant + \sum_{i=1}^{n} \left(\lambda_i \frac{\tau_{ij}^{\eta}}{\tau_i^B} - \frac{\kappa_i}{2} \left(\frac{\tau_{ij}^{\eta}}{\tau_i^B} - 1 \right)^2 \right) \tag{7}$$

where λ_i is the marginal benefit of increasing the relative signal precision (de-biasing), which depends on the aggregate posterior variances and the common de-biasing activities. Importantly, λ_i does not depend on investor j's decision, since any investor is atomic and cannot affect the aggregate posterior variances. Then the optimization problem is quite straightforward: each skilled investor chooses an optimal level of de-biasing b_{ij}, or equivalently, the relative signal precision $\theta_{ij} \equiv \frac{\tau_{ij}^{\eta}}{\tau_i^B}$, for each stock to maximize her utility in Equation 7. We reach the following lemma on optimal information acquisition.

Lemma 2. *In the equilibrium, each investor j chooses the same optimal signal precision for a factor i as follows*

$$\tau_{ij}^{\eta} = \tau_i^B \left(1 + \frac{\lambda_i}{\kappa_i} \right) \tag{8}$$

Equivalently, the optimal de-biasing level is given by

$$b_{ij} = 1 - \sqrt{\frac{\kappa_i}{\lambda_i + \kappa_i}} \tag{9}$$

where

$$\lambda_i = \frac{1}{2\gamma\sigma_i^B} \left(\bar{\sigma}_i + \gamma^2 \bar{\sigma}_i^2 x_i^2 \right) \tag{10}$$

Proof. See Appendix C ∎

[8] We denote the first as as the precision channel and the second as the de-biasing channel. Our analysis focuses specifically on the de-biasing channel.

The optimal level of signal precision features two channels: the benefit channel λ_i and the cost channel κ_i. A higher λ_i (κ_i) leads to higher (lower) de-biasing of stock i.

Corollary 3. *The marginal benefit of increasing relative signal precision, λ_i, is decreasing in the de-biasing level, b_i.*

Proof. See Appendix C □

Greater levels of de-biasing reduces the posterior variance, which is positively related to the benefit of de-biasing. Intuitively, once a signal has been sufficiently de-biased, additional de-biasing yields little benefits. Theoretically, this ensures the existence of an interior optimal level of de-biasing, as the marginal cost of de-biasing increases with the de-biasing level.

The equilibrium is such that all investors choose the same de-baised signal precision τ_{ij}^{η} following Equation 8. In addition, λ_i is determined by investors' aggregated signal precision. In the symmetric equilibrium, the relative signal precision θ_i^* for factor i is characterized by the fixed-point problem below

$$f(\theta_i) \equiv \kappa_i (\theta_i - 1) - \lambda_i = 0 \tag{11}$$

2.1.3 Implications

Uncertainty and Return Predictability In this subsection, we derive formal propositions that present the two channels (the *cost channel* and the *benefit channel*) through which prior uncertainty affect information acquisition (captured by return predictability of analysts' bias). Intrinsic uncertainty, which manifest in the cross-section of firms, and temporal uncertainty, which varies across time, impact information acquisition in opposite ways. The cost channel is driven by *intrinsic uncertainty*, and the benefit is driven by *temporal uncertainty.*

Proposition 1 (**Intrinsic uncertainty and return predictability**)**.** *In the equilibrium, a higher intrinsic uncertainty σ_i^F lowers de-biasing activity (less information acquisition) and increases return predictability of analysts' forecast biases*

Proof. See Appendix C □

First, Proposition 1 shows a negative relation between intrinsic uncertainty and de-baising activity. This is due to the fact that higher intrinsic uncertainty leads to higher information costs, which suppresses de-biasing and information acquisition.

In our model, intrinsic uncertainty affects de-biasing through both the cost and benefit channels: First, higher intrinsic uncertainty increases the marginal benefit of acquiring information, as shown in the proof, $\frac{d\lambda_i}{d\sigma_i^F} > 0$. This encourages investors to de-bias more and obtain a more precise signal. Second, a higher intrinsic uncertainty increases the marginal cost of acquiring information, leading to lower de-biasing. In our model, the cost channel always dominate the benefit channel. This is because an increase in σ_i^F leads to a proportional increase in the marginal cost of information acquisition, whereas its effect on the marginal benefit operates through the prior uncertainty, which σ_i^F only partially influences. In other words, the impact of an additional unit increase in σ_i^F on marginal cost is greater than its effect on marginal benefit, leading to the dominance of the cost channel.

Second, Proposition 1 presents a positive relation between intrinsic uncertainty and return predictability. The reason is that a higher intrinsic uncertainty discourages de-biasing and information acquisition, thus price and return are more sensitive to the analyst forecast biases.

Proposition 2 (**Temporal uncertainty and return predictability**). *In the equilibrium, a higher temporal uncertainty σ^S increases de-biasing activity (more information acquisition) and decreases return predictability of analysts' forecast biases*

Proof. See Appendix C □

Proposition 2 shows that information acquisition increases with temporal uncertainty. When temporal uncertainty increases (e.g., due to heightened macroeconomic uncertainty) and the that intrinsic uncertainty/information cost are sticky, the marginal benefit of de-biasing increases, thus investors de-bias more and return predictability of bias becomes weaker.

Overall, Proposition 2 presents a negative correlation between the temporal uncertainty and information acquisition/bias return predictability. We summarize this channel as the benefit channel.

We then reach a contrasting relation between posterior volatility and return predictability in the cross-section versus in the time-series. Figure 3 shows an numerical example of the de-biasing level and return predictability as a function of the two types of uncertainty in the equilibrium. The figure shows the contrasting influence of the temporal uncertainty and intrinsic uncertainty, consistent with Propositions 1 and 2.

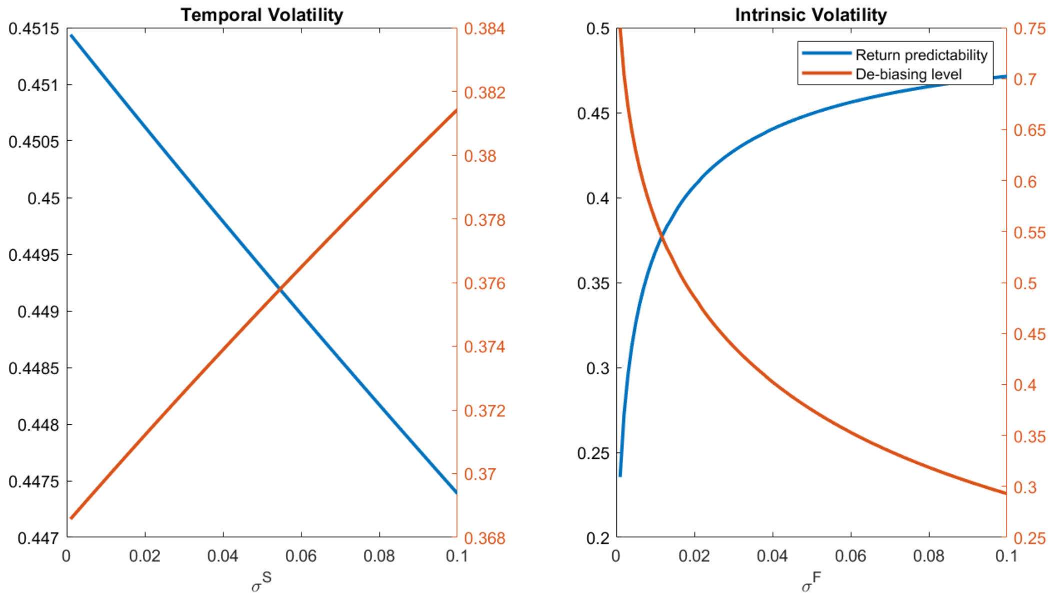


Figure 3: **Return predictability and de-biasing vs. intrinsic and temporal uncertainty.** The numerical exercise is implemented by setting $\sigma^F = 0.05$ in the right panel, $\sigma^S = 0.05$ in the left panel, and $x = 1$, $\gamma = 2$, $\psi = 1$, and $k = 1$ in both panels.

Fundamental Anomalies and Price Efficiency In addition to the implications regarding the return predictability of analyst forecast bias. Our model can also shed light on the effects of intrinsic and temporal risk on the price efficiency and fundamental/accounting-based anomalies.

Specifically, We define the price efficiency as the price sensitivity to the fundamental shock A_z, and the fundamental-based return anomaly as the return sensitivity to the fundamental shock ζ_i^z. Then we generate the following propositions.

Proposition 3. *In equilibrium, a higher intrinsic (temporal) uncertainty decreases (increases) the extent to which the price reflects the fundamental shock, $A_{z,i}$, and thus increases (decreases) the fundamental-based return anomaly, as measured by the return predictability of the fundamental shock, ζ_i^z.*

Proof. See Appendix C □

Proposition 3 shows that when intrinsic uncertainty increases, investors have less incentive to process analysts' forecasts and correct biases due to the higher cost of information. As a result, investors receive a less precise signal, and the equilibrium price reflects less of the innovations in fundamental information. Since future returns are determined by the difference between the fundamental and the price, alternative proxies for innovations in fundamentals—such as accounting variables—predict returns. Supporting this interpretation of return predictability by proxies for fundamentals, Hirshleifer and Ma (2024) find that the introduction of new technologies that reduce information-processing costs leads to a decrease in mispricing related to accounting-based anomalies.

Conversely, when temporal uncertainty increases, the marginal benefit of processing information rises. As the cost of information remains sticky, investors are more inclined to debias, and the relative precision of the signal improves. As a result, prices become more reflective of fundamental shocks, and return predictability by fundamental variables weakens.

2.2 Testable Predictions

Our model leads to testable predictions concerning the cross-sectional and time-series relationships between information processing costs, uncertainty, and investors' information acquisition. First, SIC implies there is a stronger correlation between measures of information costs and uncertainty across firms rather than over time. We test the predictions using direct measures of information processing costs proposed in the literature.

Second, the model implies a negative cross-sectional relationship between information processing costs and information acquisition. We test this prediction using using the return predictability of analysts' ex-ante biases and our measure of information costs.

Third, the model predicts contrasting relationships between investors' information acquisition and uncertainty as it relates to temporal uncertainty (variations in uncertainty across time) and intrinsic uncertainty (variations in uncertainty across firms). Specifically, SIC implies that the information cost channel plays a more dominant role in shaping the relationship for intrinsic uncertainty than in shaping the relationship for temporal uncertainty. In the case that the effect of the information cost channel is large enough, we could observe an opposite relationship between information acquisition and variations in temporal uncertainty relative to intrinsic uncertainty. We test these predictions using the return predictability of analysts' ex-ante biases and multiple measures of uncertainty.

To better illustrate this implication, Figure 4 shows the contrasting relation between posterior volatility and information acquisition. Both are driven by exogenous prior uncertainty. However, while posterior uncertainty is always increasing with the two types of prior uncertainty, information acquisition is affected by the two types of uncertainty with opposite directions.

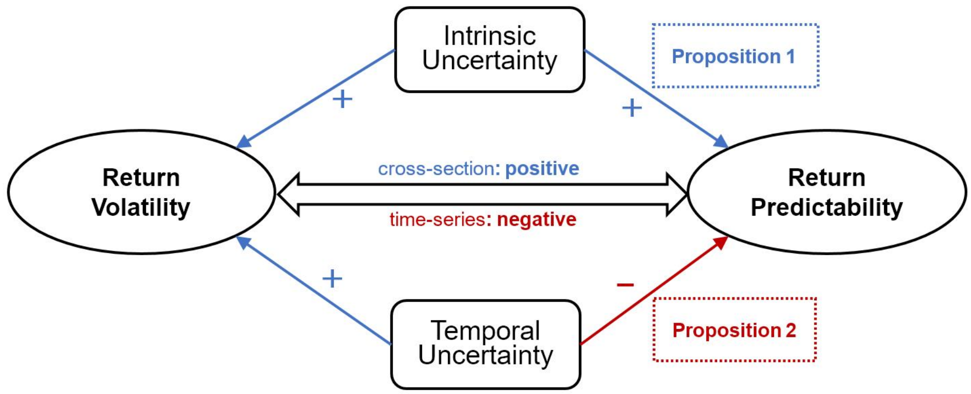


Figure 4: **The contrasting relationship between return volatility and return predictability.**

Finally, the model provides a new perspective on the variations in return anomalies. It predicts that the return predictability of ex-ante biases is weaker among large-cap stocks for which information costs are lower. It also predicts that return anomalies related to investors' inefficient processing of earnings-related information would also exhibit opposite relationships between temporal and intrinsic uncertainty.

3 Data and Measurement

Our sample consists of U.S. common stocks that are covered in the intersection of CRSP, Compustat, and I/B/E/S. We exclude micro-cap stocks, defined as stocks with a market capitalization below the NYSE 20th percentile, and low-price stocks, defined as stocks with a price below $5.

3.1 ML-based Earnings Forecasts

We construct the statistically optimal earnings forecasts, following the recommended machine learning (ML) specification in Campbell et al. (2023).[9] Similar to van Binsbergen et al. (2022), we compute the ex-ante measure of the conditional biases in analysts' forecasts

[9] Campbell et al. (2023) provides a detailed review of the machine learning earnings-forecasting literature. The recommended machine learning specification is similar to those used in van Binsbergen et al. (2022); de Silva and Thesmar (2022).

as the difference between analysts' forecasts and the ML forecasts, which we refer to as ex-ante human bias (EHB). Our EHB measure is the weighted average of one-year-ahead and two-year-ahead EHBs such that the weighted distance from the current month to the fiscal period end is a constant 12 months.[10] Appendix A provides a detailed description of the construction of EHB. We use the return predictability of EHB to quantify the extent to which investors unravel the predictable errors in analysts' forecasts, as discussed in the introduction.

Our ML earnings forecasts begin in June 1990 as the forecasts require sufficient data in the training sample. As a result, our final sample period is from June 1990 through December 2019. We provide detailed variable definitions in Table 1.

3.2 Measures of Information Costs

We follow the prior literature in finance and accounting (e.g., Begenau et al., 2018; Blankespoor et al., 2020) to construct direct measures of information cost. The literature indicates that information complexity and scarcity are two significant factors influencing information cost. Intuitively, firms with more complex disclosures and less readily available information necessitate higher information processing costs to de-bias analysts' forecasts.

To measure information complexity, we use the Bog index and the log net file size of 10-Ks, following the methodologies of Bonsall et al. (2017); Loughran and Mcdonald (2014, 2016).[11] The Bog index captures the plain English attributes of 10-K statements, focusing primarily on the writing clarity in firms' disclosures. In contrast, the log net file size provides a simple and effective gauge of the overall complexity of the firm. As Loughran and Mcdonald (2016) argue, the readability of 10-Ks and the business complexity are ultimately intertwined, so we employ both measures jointly to capture information complexity.

To measure firm-level differences in information scarcity, we use firm age, which is the number of months since the first trading day for each firm. The idea is that as a firm ages, more information becomes available for investors to analyze its fundamentals. As an example of firms' fundamental information, IBM (which had its IPO well before EDGAR came into

[10] For example, if in month t, the firm is 6 months from the one-year-ahead fiscal period end and therefore 18 months from the two-year-ahead fiscal period end, our composite EHB measure would weight each individual EHB by 0.5. Additionally, we require the FY2 forecast to be non-missing. Our results are robust to alternative specifications of the composite EHB such as the average across the one-quarter-, one-year-, and two-year-ahead EHB measures.

[11] Bonsall et al. (2017); Loughran and Mcdonald (2014, 2016) show that the Bog index and the net file size are superior measures for capturing information complexity than the Fog index. We download these measures directly from their respective websites.

existence) had 105 10-K and 10-Q filings from the beginning of EDGAR through the end of 2020, whereas Tesla, which filed its IPO in 2010, had only 42 up to that date.

We recognize that each of the three measures may contain measurement errors. To address this concern, we construct an information-cost index (IC index) that integrates the three measures. Specifically, at the end of June of each year, we first orthogonalize the cross-sectional normalized rank of each measure (Bog index, log net file size, and firm age) against the cross-sectional normalized rank of Size (Market Capitalization) to control for the impact of firm size. We then average the residuals of these regressions to create the information-cost index. This measure is applied from June of year t to May of $t+1$.

These information cost measures exhibit high persistence over time and large cross-sectional variations across firms. First, we regress the measures on their one-year lagged values. The regression coefficients are 0.88 for firm age, 0.92 for the Bog index, and 0.65 for net file size, which correspond to a half-life of 5.42, 8.31, and 1.61 respectively.[12] These results are consistent with the notion that firm-level information processing costs evolve slowly over time.

3.3 Uncertainty

3.3.1 Temporal Uncertainty

We select three measure for temporal uncertainty. First, we use the Economic Policy Uncertainty (EPU) measure provided by Baker et al. (2016).[13] As a second measure, we use the common idiosyncratic volatility (CIV) factor proposed in Herskovic et al. (2016).[14] These are both backward-looking aggregate uncertainty measures. Finally, we use the forward-looking macroeconomic uncertainty (MU) provided by Jurado et al. (2015) and Ludvigson et al. (2021). We prefer MU to the VIX index as the forward-looking aggregate uncertainty measure because VIX is also affected by risk premia. The use of both forward-looking measures of uncertainty help to validate that our results are not driven by realized measures of uncertainty.

[12]We conduct this analysis annually as the Bog index and Net File Size are updated annually with the 10-K and Firm Age is slow moving. These results are shown in Table A3 of Appendix B.

[13]According to Baker et al. (2016), EPU "capture(s) uncertainty about who will make economic policy decisions, what economic policy actions will be undertaken and when, and the economic effects of policy actions (or inaction)."

[14]For our analysis, we de-mean CIV across our sample period.

3.3.2 Intrinsic Uncertainty

We consider three measures of intrinsic uncertainty. First, we consider idiosyncratic volatility (IVOL). Secondly, we use a measure derived from regressing, by firm, IVOL on CIV over a 36 month rolling window and obtaining the constant. We denote this as IVOL Orthogonal. This allows us to capture the remaining/firm specific IVOL after removing the common idiosyncratic volatility factor. Finally we also use option implied volatility (OIV). This allows us to evaluate our results using a backward-looking, or realized, measure as well as a forward-looking measure of uncertainty. As an alternative measure of IVOL Orth. and CIV, we also we also dissect firm-level IVOL into two distinct components: a persistent component that captures cross-firm differences in intrinsic uncertainty and a time-series variation component that captures temporal fluctuations in uncertainty. Empirically, we use a firm's rolling average of IVOL over the past 36-months, ("IVOL_{MA36}") to proxy for the former and the ratio between the current value of IVOL and the persistent component ("Abnormal IVOL") to proxy for the latter.

4 Empirical Results

4.1 Information Processing Costs and Uncertainty

Based on measures of information scarcity and complexity described above, we being by examining the first of the testable predictions generated by the model that suggests that a stronger correlation between information processing costs and intrinsic uncertainty rather than temporal uncertainty. We examine these relations between information costs and uncertainty using two tests.

In our first test, we regress one measure of intrinsic uncertainty (IVOL) on our measures of information costs, controlling for firm size as well as firm or time fixed effects. Regressions with the time fixed effects capture the cross-sectional relation between uncertainty and information costs whereas those with firm fixed effects capture the time-series correlations between uncertainty and information costs. As Table 2 shows, the coefficient estimates associated with the cross-firm relation (Columns 2, 4, 6) are consistently higher than those with the time-series relation (Columns 1, 3, 5).[15] Furthermore, the statistical significance is consistently stronger for the cross-sectional relation (with time fixed effects) than for the time-series relation.

[15] We show that these results are robust by using Option Implied Volatility (OIV) in Appendix Table A4

In our second test, we evaluate the correlation between the information cost measures and our measures of temporal and intrinsic uncertainty, as shown in Figure 5. Our model suggests that the the correlation between the information measures and the persistent component should be more positive than the correlation with the temporal component. As Figure 5 shows, the IC Index, along with its constituent measures, all show a positive correlation with our various intrinsic uncertainty measures: IVOL, IVOL_{MA36}, IVOL Orthogonal, and OIV. Conversely, the temporal uncertainty measures–Abnormal IVOL, CIV, EPU, and MU–all have a slightly negative correlation with the information cost measures. These results further confirm SIC.[16]

In summary, our results in this subsection support the prediction of Assumption 1 of the model that information costs are more strongly correlated with persistent differences in intrinsic uncertainty than with differences in temporal uncertainty.

4.2 Information Processing Costs and Investors' Information Acquisition

Our model predicts an unambiguously negative relationship between information costs and investors' information acquisition: as information costs increase, investors acquire less information, thus de-biasing analysts forecasts less. Therefore, we should observe the return predictability of EHB (the negative of the information acquisition) to be stronger among firms with higher information costs.

To test the relationship, we first sort stocks into terciles according to the IC Index, and within each IC Index tercile, we further sort stocks into quintiles based on EHB. Table 3 shows the Fama-French Five-Factor (FF5) alphas of these 15 portfolios. In alignment with models's prediction, these results show that the return predictability of EHB increases with information costs, as measured by the IC Index. Specifically, for firms with the highest information costs (IC Index T3), the long-short portfolio based on EHB (EHB Q1-Q5) generates a monthly abnormal return of 1.054% (t-stat = 3.23). This monthly abnormal return declines to 0.752% (t-stat = 2.34) for IC Index T2 and finally to 0.226% (t-stat = 0.78) for IC Index T1 .

These results confirm our model's prediction of a negative relationship between information cost and information acquisition. Firms with greater information costs produce

[16]Table A1 in Appendix B shows that these results are robust to using option implied volatility measures comparable to CIV, IVOL Orth., denoted COIV and OIV Orth., respectively, as well as Abnormal IVOL and IVOL_{MA36}. Additionally, as IVOL Orth. and IVOL_{MA36} and Abnormal IVOL capture a similar decomposition of IVOL as IVOL Orth. and CIV, for the remainder of the results, only IVOL Orth. and CIV will be presented in the main tables. IVOL_{MA36} and Abnormal IVOL will appear in robustness tables in Appendix B.

abnormal returns that are significantly larger than firms with lower information costs.

4.3 Investors' Information Acquisition and Uncertainty

Proposition 1 suggests that the in situations where the impact of the information cost channel is smaller (i.e. over variations in temporal uncertainty), investors will increase their information acquisition when uncertainty is high. In contrast, Proposition 1 suggests when the impact of information cost channel is larger (i.e. over variations in intrinsic uncertainty), investors will reduce their information acquisition when uncertainty is high.

As investors increase (reduce) their information acquisition, they debias analyst forecasts more (less), which would lead to smaller (larger) abnormal returns. As such, we evaluate the patterns in abnormal returns as uncertainty varies.

4.3.1 Temporal Uncertainty

We first consider the three measures of temporal uncertainty. As we expect the information benefit channel to dominate when temporal uncertainty is high, we predict abnormal returns to be the lowest during high uncertainty periods. We follow a similar sorting method in Table 3. Table 4 shows how the return predictability of EHB varies across variations in temporal uncertainty using EPU, CIV, and MU. All three panels show a consistent pattern where the return predictability of EHB is weakest during high-uncertainty periods even though MU captures forward-looking uncertainty while EPU and CIV capture different measures of prevailing uncertainty.

Panel A of Table 4 shows that the EHB Q1-Q5 portfolio generates average abnormal returns of 1.261% per month (t-stat = 3.68) when temporal uncertainty using EPU is lowest ("EPU T1"), 0.803% per month (t-stat = 2.52) when EPU is the second lowest ("EPU T2"), and only 0.294% per month (t-stat = 0.97) when EPU is the highest. Moreover, the difference between the two long-short portfolio returns in EPU T1 and T3 is statistically significant and positive, amounting to 0.967 percentage points per month.

Consistent with Panel A, Panel B reveals weaker return predictability during periods of higher CIV. Specifically, EHB Q1-Q5 generates abnormal returns of 0.732% in the bottom tercile of CIV, which is statistically significant. In contrast, during periods when CIV is in the top tercile, return predictability decreases to 0.152% and becomes statistically insignificant. The spread between these two levels is 0.627%.

Finally, Panel C shows that, for MU T1 and T2, the long-short portfolio based on EHB

("EHB Q1-Q5") generates an average abnormal return of 0.692% and 0.377% per month, both statistically significant. In contrast, the long-short portfolio abnormal returns for MU T3 (i.e., periods when MU is highest) are only 0.018% and statistically insignificant. These findings show that our results are not driven by using forward or backward measures of uncertainty.[17]

Overall, this table supports the benefit channel outlined in the model, consistent with the prediction that higher temporal uncertainty enhances the benefit of de-biasing, which leads to lower return predictability.

To the best of our knowledge, we are the first to document systematic variations in EHB return predictability relative to temporal uncertainty. These results indicate that investors acquire more information to de-bias analysts' forecasts during periods of higher uncertainty, which is consistent with the information benefit channel being the dominant force of investors' information choice when time-series uncertainty is high. Our results thus corroborate prior findings in Bonsall et al. (2020); Benamar et al. (2021); Hirshleifer and Sheng (2022), supporting the important role of the information benefit channel in explaining the relation between uncertainty and information acquisition.

4.3.2 Intrinsic Uncertainty

Proposition 1 of the model predicts that for firms with high intrinsic uncertainty, the information cost channel dominates, suggesting that abnormal returns will be greatest for firms with high intrinsic uncertainty. Table 5 shows the variations in return predictability across our measures of intrinsic uncertainty: IVOL, IVOL Orth., and OIV. Consistent with our predictions, each measure of intrinsic uncertainty display a consistent pattern where the return predictability of EHB is strongest for high-uncertainty firms.

Panel A of Table 5 shows that the when using IVOL as the measure of intrinsic uncertainty, the EHB Q1-Q5 portfolio generates average insignificant abnormal returns of 0.254% per month when for firms with the lowest uncertainty and increases to 1.551% per month (t-stat = 3.85) when IVOL is the largest. This generates a difference in the two long-short portfolio abnormal returns of -1.297% (t-stat = -3.84) that is statistically significant.

Panels B and C of Table 5 generate similar results with the EHB Q1-Q5 for firms in the lowest uncertainty tercile generating insignificant abnormal returns of 0.102% and 0.233% for IVOL Orth. and OIV, respectively. For firms with the highest uncertainty, these abnormal

[17] Table A5 in the Appendix B shows consistent results using Abnormal IVOL and COIV.

returns grow to significant values of 1.035% and 1.456% using IVOL Orth. and OIV.[18]

Overall, Table 5 provides evidence of the model's prediction related to the information cost channel. For firms with high intrinsic uncertainty, it is more costly to acquire information needed to de-bias the analysts' forecasts relative to firms with low intrinsic uncertainty. This leads to greater return predictability for firms with high intrinsic uncertainty.

4.3.3 Contrasting Patterns

We tie our analysis together using Figure 6. The figure provides graphical evidence of the EHB Q1-Q5 abnormal returns across the terciles for the temporal and intrinsic uncertainty measures from Tables 4 and 5. This figure provides visual evidence that there is a distinctive and contrasting pattern in the return predictability across variations in temporal and intrinsic uncertainty. As suggested by our model, when intrinsic uncertainty, as proxied by IVOL, IVOL Orth, and OIV, is high, the information cost channel should dominate. We show that the abnormal returns are greatest for firms with the largest values of each uncertainty measure. Conversely, when temporal uncertainty is high, the information benefit channel is expected to dominate. In periods of the highest uncertainty across our sample, as proxied by EPU, CIV, and MU, we show that abnormal returns are the smallest.[19]

4.3.4 A Broader Set of Anomalies

Embedding the predictions of the model into information choice theories also provides a new perspective on a broader set of variations in return predictability.

4.3.5 Variation of Return Predictability of EHB across Firm Size

Information choice theory provides a new perspective regarding why the return predictability of EHB should be weaker among larger firms. From the information benefit channel, de-biasing analysts' forecasts provides more benefit to investors as larger firms account for a larger share of investors' total wealth; from the information cost channel, big firms produce more data and therefore have reduced information processing costs of investors relative to those of smaller firms (Begenau et al., 2018). Therefore, the information benefit

[18]Table A5 in the Appendix B shows consistent results using IVOL_{MA36} and and OIV Orth.

[19]Table A2 in Appendix B show that the contrasting patterns also hold when using IVOL_{MA36} and Abnormal IVOL as well as OIV Orth. and COIV.

and cost channels both predict that the return predictability of EHB should decrease with firm sizes.

Table 6 presents evidence supporting this prediction. We examine the return predictability of EHB among different size segments based on NYSE breakpoints. Consistent with the hypothesis that firm size correlates with investors' information processing costs, EHB return predictability decreases monotonically in Size. The long-short EHB portfolio (EHB Q1-Q5) has the highest abnormal return among small-cap stocks, yielding 0.968% per month (t-stat = 4.16). The abnormal return declines to 0.700% per month (t-stat = 2.85) for large caps and 0.308% per month (t-stat = 1.55) for the mega-caps. The difference in abnormal returns between small- and mega-cap stocks is 0.660 percentage points per month (t-stat= 3.94), which is economically significant. Therefore, our results indicate that information choice theory not only explains the long-standing puzzle of why investors do not fully unravel analysts' bias but can also explain our novel finding that this return predictability concentrates among non-mega-cap stocks.[20]

4.3.6 The Relation Between Uncertainty and Earnings Related Anomalies

Besides the size effect, we show that the predictions of the model holds for announcement-day returns (Bernard and Thomas, 1990) and analysts' forecast revisions (Givoly and Lakonishok, 1980).

We adopt the same portfolio sorting methodology as in Tables 4 and 5 and show the results for the announcement-day returns and analysts' forecast revisions in Tables 7 and 8, respectively. Notice that we divide the results here separate analyses for mega- and non-mega-cap stocks driven by our discussion in the previous subsection. Consistent with the pattern we find based on EHB, the return predictability associated with both variables is, on average, positively related with the persistent, variations in intrinsic uncertainty as measured by IVOL, IVOL Orth., or OIV, while being simultaneously negatively related with the variations in temporal uncertainty as measured by EPU, CIV, and MU.[21] These results are consistent with our hypothesis that the information cost (benefit) channel is the dominant driver of the relation between the extent to which investors obtain and process information for variations in intrinsic (temporal) uncertainty. We note stronger results for

[20] We present our results from Figure 6 for the mega-cap and non-mega cap-firms in Table A6 in Appendix B and show that while our results are generally consistent in both subsets. The results are stronger in the non-mega-cap firms.

[21] Tables A7 and A8 show similar results using IVOL_{MA36}, Abnormal IVOL, OIV Orth., and COIV.

variations in intrinsic uncertainty relative to temporal uncertainty. Given that these two variables have been shown to be persistent and robust predictors for future returns and are able to price a broad set of asset returns (Daniel et al., 2020; Kothari et al., 2016), our novel results regarding their distinct variations with variations in temporal and intrinsic uncertainty provide another piece of evidence supporting the information choice perspective of return predictability. Next, we evaluate alternative theories proposed in the literature for explaining our empirical findings.

4.3.7 Alternative Explanations

Existing theories of return predictability emphasize the role of risk exposures, behavioral biases, and limits of arbitrage. In this section, we explore whether these theories explain our key empirical finding–the contrasting relationship between uncertainty and the degree to which investors efficiently process analysts' forecasts in the cross section versus in the time series.

First, risk-based theories might account for the contrasting cross-sectional and time-series correlations between uncertainty and EHB return predictability if the risk exposures of the EHB Q1-Q5 long-short portfolio relate oppositely to uncertainty across these two dimensions. However, no theoretical model to date has posited such a mechanism. Empirically, if the FF5 model accurately reflects appropriate risk exposures, our results, which are based on the FF5 alphas, imply that risk-based theories fall short of explaining our empirical finding.

Second, one explanation grounded in behavioral biases is that analysts' biases might correlate positively with cross-sectional fluctuations in uncertainty, yet negatively with time-series fluctuations. However, most behavioral theories (e.g., Hirshleifer, 2001) propose an unambiguously positive link between uncertainty and human biases. Empirically, we can test this explanation by regressing the magnitude of our measure of analysts' bias (i.e., EHB) on uncertainty measures. The first two columns of Table 9 show the results. We observe a positive correlation between analysts' bias and both cross-sectional and time-series variations in uncertainty—consistent with behavioral theories but not supporting this explanation.

Third, perhaps investors' attention correlates negatively with cross-sectional variations but positively with time-series variations in uncertainty. In information choice theories, investors' attention typically has a constant marginal cost, thus its variations are completely driven by information benefits. In behavioral theories, the relationship between uncertainty and attention is complex and depends on whether uncertainty either diverts or draws attention. Empirically, we directly test this possibility by regressing a measure of attention (human downloads from EDGAR from Ryans (2017)) on uncertainty measures. Contrary to the explanation based on attention, we find that EDGAR downloads are positively rather than negatively related to cross-sectional variations in uncertainty.[22]

Finally, theories based on limits of arbitrage could rationalize the contrasting relationship if trading costs are positively associated with cross-sectional variations but negatively with time-series variations in uncertainty. Contrary to this explanation, micro-structure theories predict an unambiguous positive relation between uncertainty and trading costs as higher uncertainty leads to increased information asymmetry and thus higher trading costs. Empirically, we directly test this explanation by regressing a trading cost measure (i.e., the effective spread) on uncertainty measures. In columns 5 and 6 of Table 9, we find that trading cost is either positively or insignificantly correlated with both time-series and cross-sectional variations in uncertainty, which aligns with micro-structure theories but contradicts this hypothesis.

In summary, alternative theories of return predictability struggle to explain the contrasting relationship between uncertainty and the return predictability of EHB. Our purpose is not to reject all variations of these alternative theories, but to underscore that the primary empirical result detailed in this paper offers a valuable empirical moment that helps distinguish information choice models from these competing theories.

5 Conclusion

This paper studies the relationship between uncertainty and investors' information acquisition decisions. While existing studies suggest that higher uncertainty increases the benefits of information acquisition, our paper focuses on how uncertainty is related to the cost of information acquisition. Our primary contribution is in highlighting that the cost of information may fluctuate differently in response to cross-sectional variations as opposed to time-series variations in uncertainty. This differential relationship is critical for understanding how uncertainty influences investors' choices regarding information acquisition.

Specifically, we introduce and test the Sticky Information Cost (SIC) hypothesis, which posits that information costs are more closely related to cross-sectional variations in uncertainty than to time-series variations. This hypothesis stems from the observation that the

[22]Tables A9 and A10 in Appendix B shows that these results are robust to using OIV as well as AIA, a measure of abnormal investor attention, from Ben-Rephael et al. (2017).

expenses incurred in processing information for investors are shaped by a firm's slow-evolving information environment. Utilizing direct measures for information processing costs and using the return predictability of analysts' biases as a proxy for information acquisition, we find opposite relationships between uncertainty and investors' information acquisition when comparing across firms versus over time. These contrasting patterns are robust to using various uncertainty measures and extend to other earnings-related anomalies, thereby lending support to the SIC hypothesis. In contrast, alternative theories struggle to offer a unified explanation of these contrasting patterns. Therefore, our results offer a novel perspective on return anomalies and underscore the importance of distinguishing between cross-sectional and time-series variations in uncertainty when modeling investors' decisions regarding information acquisition.

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Figure 5: Correlation Matrix of Information Cost Index Components and IVOL Components

This figure shows the Spearman correlation matrix for the components of the Information Cost Index and the components of IVOL. As the Information Cost Index consists of measures that update infrequently (the Bog Index, Complexity, and Net File Size update annually and Firm Age is slow moving), the analysis is done as of the end of June in each year. The Information Cost Index is the average of the residual of the normalized rank (i.e., the rank scaled by the number of stocks in the cross section) of -LN(Firm Age), the Bog Index, and LN(Net File Size), each orthogonalized to the normalized rank of Size. For comparability, the normalized rank of the IVOL related variables (IVOL, IVOL_{MA36}, and Abnormal IVOL, IVOL Orth.) are also orthogonalized to the normalized rank of Size. IVOL Orth. is calculated by regressing IVOL on CIV (demeaned) using a rolling 36-month window. As CIV does not vary across a given month, CIV related correlations are calculated by firm and then averaged. The sample period for Firm Age and IVOL (and its components) begins in June 1990, the annual sample period for the Bog Index and Net File Size begins in June 1996. All samples end in June 2019.

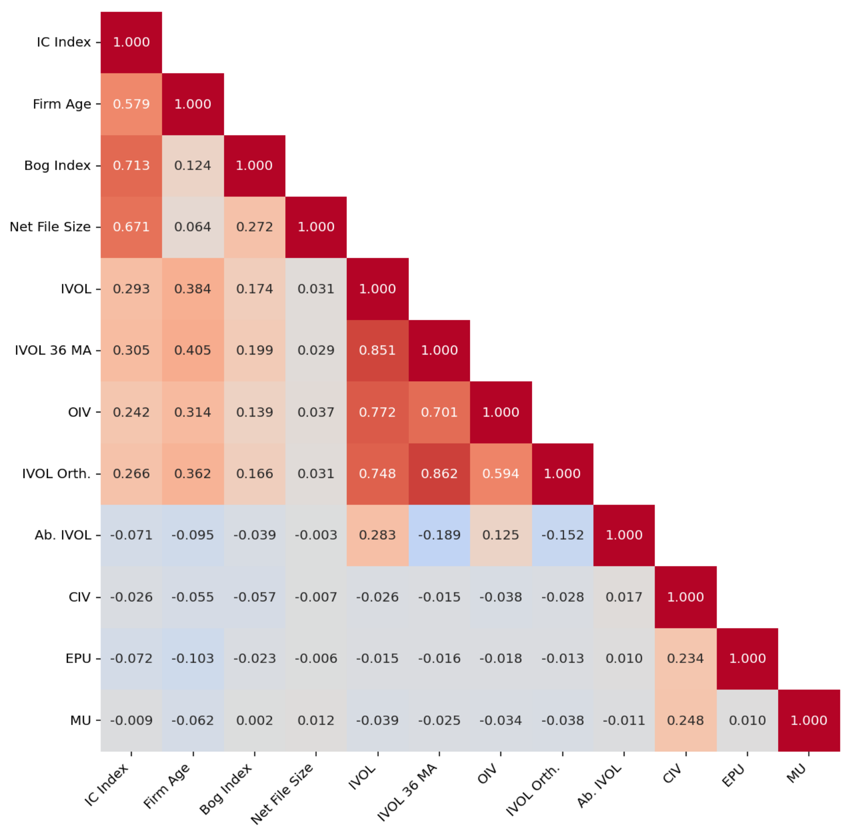

Figure 6: Return Predictability of EHB by Uncertainty Terciles

This figure presents the Fama-French Five-Factor alphas for double-sort portfolios created by first sorting companies into terciles based on various uncertainty measures, then conditionally cross-sectionally sorting into quintiles based on EHB. Only the EHB Q1-Q5 alpha is shown for each tercile. The EHB Q1-Q5 portfolios based on IVOL, IVOL Orth., and OIV are made by first cross-sectionally sorting companies into terciles based on the specific uncertainty measure. The EHB Q1-Q5 portfolios based on EPU, CIV, and MU are made by sorting observations into terciles in the time series based on each uncertainty measure. Portfolios are value weighted and are re-balanced monthly. Q1 (Q5) contains firms with the lowest (highest) values of EHB. T1 (T3) of each uncertainty tercile contains firms with the lowest (highest) values. Whiskers denote 95% confidence bands. Standard errors of the resulting regression coefficients are computed based on Newey and West (1987) with 12 lags. The sample period is from June 1990 to December 2019, with the exception of OIV which begins in January 1996.

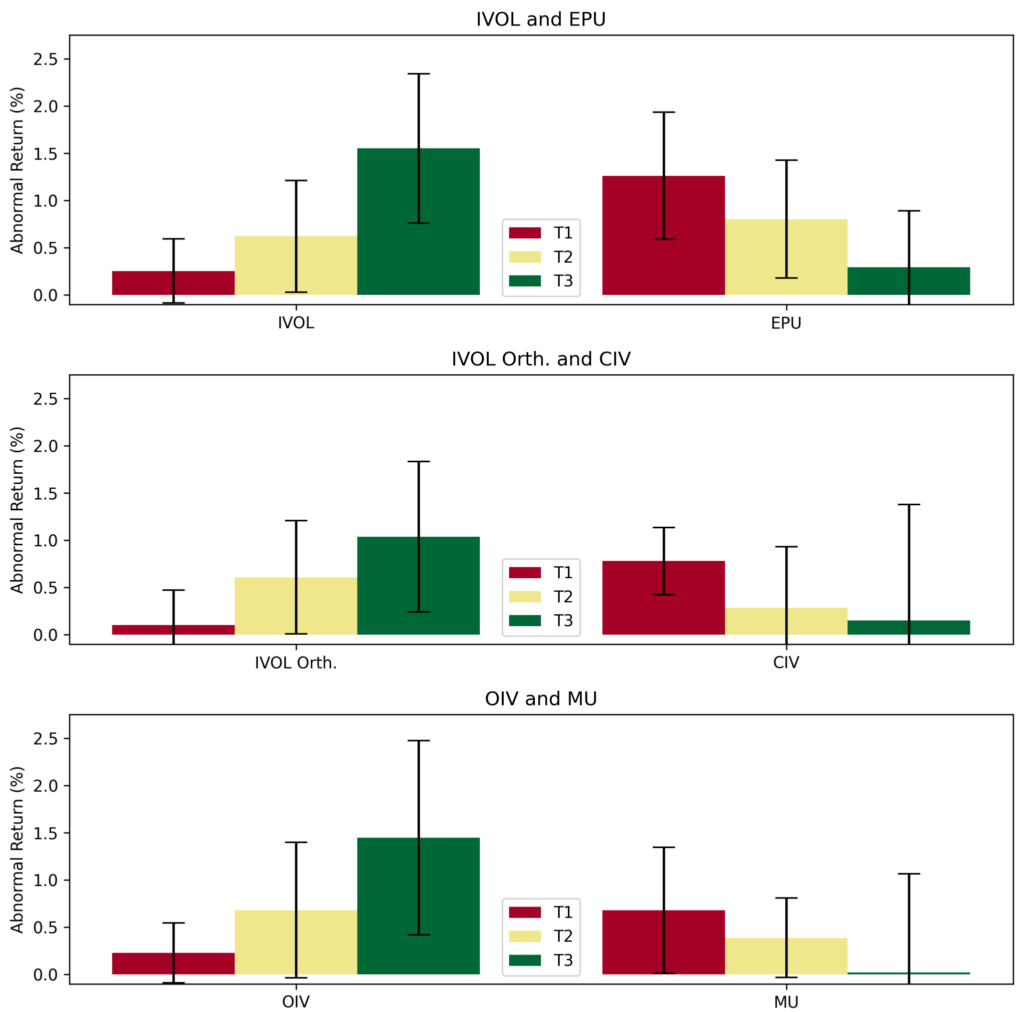

Table 1: Key Variable Definitions

This table provides the definition of key variables in the analysis.

Variable	Definition
Ex-Ante Human Bias (EHB)	Analysts' conditional biases (Analysts' forecast − ML Model forecast) (Constant 12 months to Fiscal Period End calculated as the weighted average of FY1 and FY2). EHB is scaled by price.
Firm Age	Firm Age (months since first trading day)
Size	Ln(Market Capitalization) (daily or as of end of month)
Mega Cap	Firms with market capitalization above 80th percentile of NYSE firm size
Small Cap	Firms below median NYSE market capitalization
Large Cap	Firms above median NYSE market capitalization but not Mega Cap
IVOL	Standard deviation of residuals from CAPM regressions using the past year of daily data. (Require at least 100 non-missing observations.)
OIV	Average of call and put option implied volatility from the volatility surface using 30-day maturity and delta=0.5 (-0.5 for put options) on last day of the month
IVOL Orth.	The constant obtained from a regression of IVOL on CIV, by firm, using a 36 month rolling window.
IVOL$_{MA36}$	Moving Average of IVOL from month $t-35$ to t (Trailing IVOL)
Abnormal IVOL	$\frac{IVOL}{IVOL_{MA36}}$
MU	One Month Macro Uncertainty Measure (Ludvigson et al. (2021))
EPU	Economic Policy Uncertainty Index (Baker et al. (2016))
CIV	Common Idiosyncratic Volatility Factor, which is the cross-sectional average of IVOL. The measure is then de-meaned in the analysis. (Herskovic et al. (2016))
Announcement Return	Sum of risk-adjusted returns from two days before an earnings announcement to one day after the announcement
Analysts' Revision	Three-month revision in analysts' forecasts for one-quarter ahead earnings forecasts
Bog Index	Plain English Readability Measure Applied to 10-Ks (Bonsall et al. (2017))
Net File Size	File size of 10-K excluding ASCII-encoded insertions, HTML, and XBRL (Loughran and Mcdonald (2014))
Information Cost Index	Average of the cross-sectional normalized ranks of 1/Age, Bog Index, and Net File Size, each cross-sectionally orthogonalized to the normalized rank of Size. The measure is created in June of year t and is then used until May of year $t+1$.
EDGAR Downloads	Count of human downloads from EDGAR for a given month (Ryans 2017)
Effective Spread	Monthly Average Effective Spread using TAQ data

Table 2: IVOL and Information Cost

This table presents the results of pooled OLS regressions of the components of the Information Cost Index (Firm Age, the Bog Index, and Net File Size) on LN(IVOL) and Size. As the Information Cost Index consists of measures that update infrequently (the Bog Index and Net File Size update annually and Firm Age is slow moving), the regressions are run as of the end of June in each year. Columns 1 and 2 use -LN(Firm Age), columns 3 and 4 use the Bog Index, and columns 5 and 6 use LN(Net File Size). Statistical significance is denoted as ***, **, and * for $p<0.10$, $p<0.05$, and $p<0.01$, respectively. Standard errors are clustered at the firm and year level. Columns 1, 3, and 5 include firm fixed effects while columns 2, 4, and 6 include time fixed effects. The annual sample period for Firm Age begins in June 1990, the sample period for the Bog Index and Net File Size begins in June 1996. All samples end in June 2019.

	-LN(Age)		Bog Index		LN(Net File Size)	
	(1)	(2)	(3)	(4)	(5)	(6)
LN(IVOL)	0.233***	1.012***	-1.471*	3.570***	-0.062	0.049**
	(3.4)	(20.8)	(-1.8)	(7.9)	(-0.9)	(2.3)
Size	-0.356***	-0.148***	2.018***	0.435***	0.141***	0.102***
	(-16.2)	(-11.4)	(4.9)	(4.4)	(5.8)	(14.3)
Cons.	-1.484***	0.021	62.943***	94.977***	11.547***	12.288***
	(-6.9)	(0.1)	(21.0)	(56.8)	(45.3)	(151.0)
Fixed Effects	Firm	Time	Firm	Time	Firm	Time
Observations	53662	54951	40398	41585	39880	41066

Table 3: IC Index and the Return Predictability of EHB

This table presents the Fama-French Five-Factor alphas for double-sort portfolios created by first sorting companies into terciles based on the IC index, then conditionally cross-sectionally sorting into quintiles based on EHB. Portfolios are value weighted and are re-balanced monthly. Standard errors of the resulting regression coefficients are computed based on Newey and West (1987) with 12 lags. Statistical significance is denoted as ***, **, and * for p<0.10, p<0.05, and p<0.01, respectively. The sample period is from June 1996 to December 2019.

	EHB Q1	EHB Q2	EHB Q3	EHB Q4	EHB Q5	EHB Q1-Q5
IC Index T1	0.120	0.067	0.075	-0.133	-0.106	0.226
	(1.01)	(0.63)	(0.61)	(-1.03)	(-0.49)	(0.78)
IC Index T2	0.426***	0.027	0.032	0.008	-0.326	0.752**
	(3.39)	(0.31)	(0.31)	(0.05)	(-1.27)	(2.34)
IC Index T3	0.368**	-0.007	-0.083	-0.269	-0.686***	1.054***
	(2.04)	(-0.07)	(-0.65)	(-1.32)	(-3.00)	(3.23)
IC Index T1-T3	-0.248	0.073	0.158	0.137	0.580***	-0.828**
	(-1.06)	(0.47)	(0.81)	(0.55)	(2.68)	(-2.31)

Table 4: Uncertainty and the Return Predictability of Analysts' Conditional Biases

This table presents the Fama-French Five-Factor alphas for double-sort portfolios created by first sorting companies into terciles based on various uncertainty measures, then conditionally cross-sectionally sorting into quintiles based on EHB. Panel A shows results using EPU, Panel B shows results using CIV and Panel C shows results using MU. The EHB Q1-Q5 portfolios are made by first sorting observations into terciles in the time series based on each uncertainty measure. Portfolios are value weighted and are re-balanced monthly. Q1 (Q5) contains firms with the lowest (highest) values of the EHB. T1 (T3) of each uncertainty tercile contains firms with the lowest (highest) values. Standard errors of the resulting regression coefficients are computed based on Newey and West (1987) with 12 lags. Statistical significance is denoted as ***, **, and * for $p<0.10$, $p<0.05$, and $p<0.01$, respectively. The sample period is from June 1990 to December 2019, with the exception of OIV which begins in January 1996.

Panel A: EPU

	EHB Q1	EHB Q2	EHB Q3	EHB Q4	EHB Q5	EHB Q1-Q5
EPU T1	0.565***	-0.073	-0.230***	-0.232**	-0.696***	1.261***
	(4.34)	(-0.68)	(-4.43)	(-2.08)	(-2.90)	(3.68)
EPU T2	0.258*	-0.042	-0.033	-0.317**	-0.544**	0.803**
	(1.81)	(-0.48)	(-0.28)	(-2.11)	(-2.40)	(2.52)
EPU T3	0.082	0.070	0.020	0.013	-0.212	0.294
	(0.76)	(0.85)	(0.24)	(0.09)	(-0.82)	(0.97)
EPU T1-T3	0.483***	-0.143	-0.250***	-0.245	-0.484	0.967*
	(2.64)	(-1.10)	(-2.69)	(-1.14)	(-1.22)	(1.84)

Panel B: CIV

	EHB Q1	EHB Q2	EHB Q3	EHB Q4	EHB Q5	EHB Q1-Q5
CIV T1	0.336***	-0.004	-0.044	-0.075	-0.443***	0.779***
	(5.10)	(-0.05)	(-0.62)	(-0.65)	(-2.92)	(4.30)
CIV T2	-0.043	0.112*	-0.020	0.031	-0.328	0.285
	(-0.33)	(1.67)	(-0.25)	(0.24)	(-1.48)	(0.87)
CIV T3	0.320	-0.181*	-0.107	-0.124	0.168	0.152
	(1.47)	(-1.70)	(-1.12)	(-0.55)	(0.36)	(0.24)
CIV T1-T3	0.016	0.177	0.062	0.049	-0.611	0.627
	(0.07)	(1.39)	(0.52)	(0.20)	(-1.23)	(0.96)

Panel C: MU

	EHB Q1	EHB Q2	EHB Q3	EHB Q4	EHB Q5	EHB Q1-Q5
MU T1	0.234*	0.021	-0.151**	-0.065	-0.458**	0.692**
	(1.82)	(0.41)	(-2.48)	(-0.46)	(-1.99)	(2.03)
MU T2	0.149	0.060	-0.023	-0.100	-0.227	0.377*
	(1.10)	(0.66)	(-0.25)	(-0.95)	(-1.30)	(1.70)
MU T3	0.165	-0.204**	0.012	0.042	0.147	0.018
	(0.97)	(-2.28)	(0.14)	(0.21)	(0.35)	(0.03)
MU T1-T3	0.069	0.224**	-0.163	-0.107	-0.605	0.674
	(0.36)	(2.20)	(-1.56)	(-0.44)	(-1.27)	(1.10)

Table 5: Cross Sectional Uncertainty and the Return Predictability of Analysts' Conditional Biases

This table presents the Fama-French Five-Factor alphas for double-sort portfolios created by first sorting companies into terciles based on various uncertainty measures, then conditionally cross-sectionally sorting into quintiles based on EHB. Panel A shows results using IVOL, Panel B shows results using IVOL Orth. and Panel C shows results using OIV. The EHB Q1-Q5 portfolios are made by first sorting observations into terciles in the time series based on each uncertainty measure. Portfolios are value weighted and are re-balanced monthly. Q1 (Q5) contains firms with the lowest (highest) values of the EHB. T1 (T3) of each uncertainty tercile contains firms with the lowest (highest) values. Standard errors of the resulting regression coefficients are computed based on Newey and West (1987) with 12 lags. Statistical significance is denoted as ***, **, and * for $p<0.10$, $p<0.05$, and $p<0.01$, respectively. The sample period is from June 1990 to December 2019, with the exception of OIV which begins in January 1996.

Panel A: IVOL

	EHB Q1	EHB Q2	EHB Q3	EHB Q4	EHB Q5	EHB Q1-Q5
IVOL T1	0.096	0.018	-0.040	-0.046	-0.159	0.254
	(1.21)	(0.24)	(-0.71)	(-0.59)	(-1.24)	(1.47)
IVOL T2	0.337**	-0.027	-0.172*	-0.199	-0.283	0.620**
	(2.29)	(-0.28)	(-1.91)	(-1.37)	(-1.30)	(2.06)
IVOL T3	0.638**	0.275	0.029	-0.418**	-0.914***	1.551***
	(2.52)	(1.62)	(0.22)	(-2.30)	(-3.57)	(3.85)
IVOL T1-T3	-0.542**	-0.256	-0.069	0.372**	0.755***	-1.297***
	(-2.14)	(-1.26)	(-0.45)	(2.05)	(3.07)	(-3.84)

Panel B: CIV

	EHB Q1	EHB Q2	EHB Q3	EHB Q4	EHB Q5	EHB Q1-Q5
IVOL Orth. T1	-0.043	0.007	-0.128**	0.008	-0.145	0.102
	(-0.50)	(0.09)	(-2.04)	(0.08)	(-0.97)	(0.54)
IVOL Orth. T2	0.369**	-0.141	-0.142	-0.180	-0.239	0.608**
	(2.45)	(-1.39)	(-1.34)	(-1.29)	(-1.00)	(1.99)
IVOL Orth. T3	0.595**	0.266**	0.052	-0.241	-0.440**	1.035**
	(1.97)	(1.99)	(0.36)	(-1.39)	(-2.08)	(2.55)
IVOL Orth. T1-T3	-0.638*	-0.259	-0.180	0.249	0.295*	-0.933**
	(-1.87)	(-1.42)	(-1.02)	(1.39)	(1.65)	(-2.53)

Panel C: MU

	EHB Q1	EHB Q2	EHB Q3	EHB Q4	EHB Q5	EHB Q1-Q5
OIV T1	0.160**	-0.009	-0.066	-0.063	-0.063	0.223
	(2.12)	(-0.10)	(-0.99)	(-0.73)	(-0.48)	(1.39)
OIV T2	0.359*	0.057	-0.176	-0.189	-0.316	0.675*
	(1.82)	(0.43)	(-1.11)	(-1.36)	(-1.29)	(1.85)
OIV T3	0.450	0.217	-0.014	-0.415*	-1.002***	1.453***
	(1.39)	(1.32)	(-0.07)	(-1.70)	(-3.63)	(2.76)
OIV T1-T3	-0.290	-0.226	-0.052	0.352	0.940***	-1.230***
	(-0.87)	(-1.08)	(-0.25)	(1.32)	(3.88)	(-2.59)

Table 6: Firm Size and the Return Predictability of EHB

This table presents the Fama-French Five-Factor alphas for double-sort portfolios created by first sorting companies into groups based on their market value of equity, then conditionally cross-sectionally sorting into quintiles based on EHB. The market value of equity groups divide the firms into mega-cap, large-cap, and small-cap groups. Mega-cap firms are defined as firms with market capitalization greater than the 80th percentile of firm sizes on the NYSE. The remaining firms are then defined as small or large cap based on whether their size is above the median NYSE market capitalization. Portfolios are value weighted and are re-balanced monthly. Standard errors of the resulting regression coefficients are computed based on Newey and West (1987) with 12 lags. Q1 indicates the lowest values and Q5 the highest values for EHB. Statistical significance is denoted as ***, **, and * for $p<0.10$, $p<0.05$, and $p<0.01$, respectively. Values are shown in percentage terms. The sample period is from June 1990 to December 2019.

	EHB Q1	EHB Q2	EHB Q3	EHB Q4	EHB Q5	EHB Q1-Q5
Small Cap	0.288***	0.225***	-0.015	-0.031	-0.681***	0.968***
	(3.13)	(3.77)	(-0.20)	(-0.33)	(-4.19)	(4.16)
Large Cap	0.319***	0.139	-0.075	-0.089	-0.381**	0.700***
	(2.71)	(1.62)	(-1.01)	(-0.89)	(-2.28)	(2.85)
Mega Cap	0.214**	0.055	-0.106*	-0.062	-0.094	0.308
	(2.26)	(0.81)	(-1.80)	(-0.94)	(-0.74)	(1.55)
Small-Mega	0.074	0.170**	0.091	0.031	-0.587***	0.660***
	(0.75)	(1.99)	(0.98)	(0.31)	(-5.48)	(3.94)

Table 7: Return Predictability of Announcement Return by Uncertainty Terciles: Mega and Non-Mega Cap

This table presents the Fama-French Five-Factor alphas for double-sort portfolios created by first sorting companies into terciles based on various uncertainty measures, then conditionally cross-sectionally sorting into quintiles based on Announcement Returns. Only the Announcement Return Q5-Q1 alpha is shown for each tercile. Panel A uses only firms in the non-mega-cap subsample, while Panel B uses only firms in the mega-cap sample. The Announcement Return Q5-Q1 portfolios based on IVOL, IVOL Orth and OIV are made by cross-sectionally sorting companies into terciles based on each uncertainty measure. The Announcement Return Q5-Q1 portfolios based on EPU, CIV, and MU are made by sorting observations into terciles in the time series based on each uncertainty measure. Portfolios are value weighted and are re-balanced monthly. Q1 (Q5) contains firms with the lowest (highest) values of Announcement Return. T1 (T3) of each uncertainty tercile contains firms with the lowest (highest) values. Standard errors of the resulting regression coefficients are computed based on Newey and West (1987) with 12 lags. The sample period is from June 1990 to December 2019, with the exception of OIV which begins in January 1996.

Panel A: Non-Mega-Cap

	IVOL	IVOL Orth.	OIV	EPU	CIV	MU
T1	0.325***	0.246***	0.310***	0.760***	0.464***	0.656***
	(3.92)	(2.78)	(3.14)	(5.39)	(4.99)	(4.96)
T2	0.455***	0.342***	0.368***	0.643***	0.556***	0.629***
	(3.88)	(2.85)	(3.00)	(3.88)	(2.63)	(3.61)
T3	1.042***	0.829***	0.575***	0.483**	0.713***	0.273
	(5.42)	(3.81)	(2.66)	(2.50)	(3.90)	(1.37)
T1-T3	-0.717***	-0.583***	-0.265	0.276	-0.249	0.384
	(-4.10)	(-2.72)	(-1.13)	(1.13)	(-1.21)	(1.57)

Panel B: Mega-Cap

	IVOL	IVOL Orth.	OIV	EPU	CIV	MU
T1	-0.021	0.010	-0.041	0.848***	0.292*	0.265*
	(-0.14)	(0.07)	(-0.31)	(3.36)	(1.79)	(1.95)
T2	0.149	0.171	0.280	0.351	0.349*	0.372*
	(1.06)	(1.06)	(1.40)	(1.46)	(1.89)	(1.90)
T3	0.815**	0.720***	0.837**	0.161	0.524	0.530
	(2.44)	(2.85)	(2.32)	(0.74)	(1.44)	(1.25)
T1-T3	-0.835***	-0.710***	-0.878**	0.687**	-0.231	-0.264
	(-2.60)	(-2.63)	(-2.04)	(2.03)	(-0.54)	(-0.66)

Table 8: Return Predictability of Analysts' Revisions by Uncertainty Terciles: Mega and Non-Mega Cap

This table presents the Fama-French Five-Factor alphas for double-sort portfolios created by first sorting companies into terciles based on various uncertainty measures, then conditionally cross-sectionally sorting into quintiles based on Analysts' Revisions. Only the Analysts' Revisions Q5-Q1 alpha is shown for each tercile. Panel A uses only firms in the non-mega-cap subsample, while Panel B uses only firms in the mega-cap sample. The Analysts' Revisions Q5-Q1 portfolios based on IVOL, IVOL Orth and OIV are made by cross-sectionally sorting companies into terciles based on each uncertainty measure. The Analysts' Revisions Q5-Q1 portfolios based on EPU, CIV, and MU are made by sorting observations into terciles in the time series based on each uncertainty measure. Portfolios are value weighted and are re-balanced monthly. Q1 (Q5) contains firms with the lowest (highest) values of Analysts' Revisions. T1 (T3) of each uncertainty tercile contains firms with the lowest (highest) values. Standard errors of the resulting regression coefficients are computed based on Newey and West (1987) with 12 lags. The sample period is from June 1990 to December 2019, with the exception of OIV which begins in January 1996.

Panel A: Non-Mega-Cap

	IVOL	IVOL Orth.	OIV	EPU	CIV	MU
T1	0.200	0.105	0.093	0.912***	0.489***	0.667***
	(1.47)	(0.62)	(0.56)	(4.29)	(3.08)	(3.78)
T2	0.426**	0.185	0.148	0.522**	0.824***	0.421**
	(2.03)	(0.67)	(0.53)	(2.33)	(2.97)	(2.00)
T3	0.684**	0.303	0.489	-0.107	-0.283	-0.101
	(2.19)	(1.22)	(1.34)	(-0.39)	(-0.58)	(-0.24)
T1-T3	-0.483*	-0.198	-0.396	1.019**	0.772	0.768*
	(-1.91)	(-0.97)	(-1.27)	(2.27)	(1.48)	(1.74)

Panel B: Mega-Cap

	IVOL	IVOL Orth.	OIV	EPU	CIV	MU
T1	-0.091	0.008	0.066	0.744**	0.554***	0.528*
	(-0.44)	(0.04)	(0.38)	(2.47)	(2.76)	(1.89)
T2	0.321*	0.008	0.285	0.570*	0.013	0.359*
	(1.65)	(0.03)	(1.34)	(1.76)	(0.05)	(1.69)
T3	1.074***	0.852***	0.890**	0.365	0.740*	0.303
	(3.74)	(4.91)	(2.30)	(1.08)	(1.95)	(1.08)
T1-T3	-1.166***	-0.843***	-0.824*	0.379	-0.186	0.226
	(-3.33)	(-3.30)	(-1.82)	(0.75)	(-0.43)	(0.58)

Table 9: Testing Alternative Theories

This table presents the results of monthly regressions of |EHB|, EDGAR Downloads, or Effective Spread on the measures of uncertainty. The |EHB| and Effective Spread analysis use pooled OLS regressions. As EDGAR Downloads are a count measure, a pseudo-Poisson regression is used instead of a pooled OLS regression. Columns 1, 3, and 5 include only Size as a control, and columns 2, 4, and 6 add Firm Age and an indicator equal to one in earnings-announcement months. Standard errors are clustered at the firm and month level. Statistical significance is denoted as ***, **, and * for p<0.10, p<0.05, and p<0.01, respectively. The sample period for |EHB| is from June 1990 to December 2019. The sample for EDGAR Downloads begins in April 2003 and ends in December 2017. The sample for Effective Spread begins in September 2003 and ends in December 2019.

	|EHB|		EDGAR Downloads		Eff. Spread	
Size	-0.167***	-0.190***	0.622***	0.634***	-3.280***	-3.237***
	(-12.1)	(-13.1)	(8.9)	(8.2)	(-22.2)	(-21.2)
EPU	0.002***	0.002***	0.004***	0.004***	-0.003	-0.003
	(3.9)	(3.7)	(4.7)	(4.7)	(-0.7)	(-0.6)
LN(IVOL)	0.651***	0.773***	0.514***	0.464***	4.467***	4.248***
	(17.8)	(18.5)	(5.9)	(8.1)	(6.7)	(6.2)
LN(Age)		0.151***		-0.081		-0.324**
		(11.4)		(-1.2)		(-2.0)
Earn. Annc.		0.054***		0.106**		0.036
		(2.8)		(2.3)		(0.1)
Observations	626852	626852	279235	279235	320627	320627

A Appendix A: EHB Construction

A.1 Calculating EHB

To create our composite EHB measure, we first generate forecasts for the next quarter (FQ), one year ahead (FY1), and two years ahead (FY2) earnings using machine learning. We then take the weighted average of the FY1 and FY2 EHB so that the average distance between the forecast and the corresponding FY1/FY2 fiscal period ends is 12 months. We require FY2 EHB to be non-missing. Additionally, we place all the weight on FY1 if the distance to the fiscal year end from the FY1 forecast is 12 months. If the distance to the FY1 fiscal year end is less than zero (i.e. the firm has had their fiscal year end, but the earnings announcement has not happened, we put a weight of zero on the FY1 forecast.Our results are robust to alternative methods of calculating EHB such as taking the average of FQ, FY1, and FY2 forecasts.

A.2 Input Dataset Construction

Tables A1 and A2 show the variables used in generating the machine learning forecasts. We utilize the methodology in Campbell et al. (2023) to generate the EHB forecasts using their suggested best specification. Please refer to their paper for a more detailed description of the data generation process.

We apply the same key filters used in Campbell et al. (2023). Specifically, we require returns, market capitalization, the two momentum variables, the current analysts' forecast, the most recently realized earnings, the stock price, and price-to-sales to be non-missing.[23] Since analysts' forecasts contain private information that adds incremental predictive power for earnings relative to financial statement variables (van Binsbergen et al., 2022; de Silva and Thesmar, 2022), we also include analysts' forecast-related variables in our predictor set as shown in Table A2.

[23] The requirement of non-missing current analysts' forecast, the most recently realized earnings, stock price, and price-to-sales follows from (Bradshaw et al., 2012).

Table A1: WRDS Financial Ratio Variables

This table provides the definitions of WRDS Financial Ratio Variables. Following van Binsbergen et al. (2022), we exclude Forward P/E to 1-year Growth (PEG) ratio, Forward P/E to Long-term Growth (PEG) ratio, Price/Operating Earnings (Basic, Excl. Extraordinary Income), and Price/Operating Earnings (Diluted, Excl. Extraordinary Income) from the WRDS Financial Suite Ratios due to the large number of missing observations.

Acronym	Definition	Acronym	Definition
accrual	Accruals/Average Assets	int_totdebt	Interest/Average Total Debt
adv_sale	Advertising Expenses/Sales	inv_turn	Inventory Turnover
aftret_eq	After-tax Return on Average Common Equity	invt_act	Inventory/Current Assets
aftret_equity	After-tax Return on Total Stockholders Equity	lt_debt	Long-term Debt/Total Liabilities
aftret_invcapx	After-tax Return on Invested Capital	lt_ppent	Total Liabilities/Total Tangible Assets
at_turn	Asset Turnover	npm	Net Profit Margin
bm	Book/Market	ocf_lct	Operating Cash Flow/Current Liabilities
capei	Shiller's Cyclically Adjusted P/E Ratio	opmad	Operating Profit Margin After Depreciation
capital_ratio	Capitalization Ratio	opmbd	Operating Profit Margin Before Depreciation
cash_conversion	Cash Conversion Cycle (Days)	pay_turn	Payables Turnover
cash_debt	Cash Flow/Total Debt	pcf	Price/Cash Flow
cash_lt	Cash Balance/Total Liabilities	pe_exi	P/E (Diluted, Excl. EI)
cash_ratio	Cash Ratio	pe_inc	P/E (Diluted, Incl. EI)
cfm	Cash Flow Margin	peg_trailing	Trailing P/E to Growth (PEG) ratio
curr_debt	Current Liabilities/Total Liabilities	pretret_earnat	Pre-tax Return on Total Earning Assets
curr_ratio	Current Ratio	pretret_noa	Pre-tax Return on Net Operating Assets
de_ratio	Total Debt/Total Equity	profit_lct	Profit Before Depreciation/Current Liabilities
debt_assets	Total Debt/Total Assets	ps	Price/Sales
debt_at	Total Debt/Total Assets	ptb	Price/Book
debt_capital	Total Debt/Total Capital	ptpm	Pre-Tax Profit margin
debt_ebitda	Total Debt/EBITDA	quick_ratio	Quick Ratio
debt_invcap	Long-term Debt/Invested Capital	rd_sale	Research and Development/Sales
divyield	Dividend Yield	rect_act	Receivables/Current Assets
dltt_be	Long-term Debt/Book Equity	rect_turn	Receivables Turnover
dpr	Dividend Payout Ratio	roa	Return on Assets
efftax	Effective Tax Rate	roce	Return on Capital Employed
equity_invcap	Common Equity/Invested Capital	roe	Return on Equity
evm	Enterprise Value Multiple	sale_equity	Sales/Stockholders Equity
fcf_ocf	Free Cash Flow/Operating Cash Flow	sale_invcap	Sales/Invested Capital
gpm	Gross Profit Margin	sale_nwc	Sales/Working Capital
gprof	Gross Profit/Total Assets	short_debt	Short-Term Debt/Total Debt
int_debt	Interest/Average Long-term Debt	staff_sale	Labor Expenses/Sales
intcov	After-tax Interest Coverage	totdebt_invcap	Total Debt/Invested Capital
intcov_ratio	Interest Coverage Ratio		

Table A2: Other Variables

This table provides the definitions of the other variables used in generating our ML predictions that are not included in the WRDS Financial Ratio Suite. EPS and ErrAF are target variables, while all other variables are additional predictors.

Acronym	Definition
EPS (FY1/FY2)	Realized earnings per share
ErrAF (FY1/FY2)	Realized EPS − Analysts' forecast as of current month
medest2	Analysts' consensus forecast for FY2 horizon
medestqtr	Analysts' consensus forecast for FQ horizon
ibes_earnings_ann	Most recently realized annual earnings as of current month
ibes_earnings_qtr	Most recently realized quarterly earnings as of current month
last_F2ana_fe_med	Most recently realized FY2 horizon analysts' forecast error as of current month
last_Fqtrana_fe_med	Most recently realized FQ horizon analysts' forecast error as of current month
rev_FY2_3m	Revision of analysts' FY2 horizon forecast between current month and 3 months prior
rev_FYqtr_3m	Revision of analysts' FQ horizon forecast between current month and 3 months prior
dist2	Distance between FY2 fiscal period end and current month
distqtr	Distance between FQ fiscal period end and current month
ret	Stock return
prc	Stock price
size	LN(Market Capitalization)
mom6m	6-month momentum
indmom	Industry weighted 6-month momentum

A.3 ML Forecast Timing

We construct our train and test datasets carefully to ensure no data leakage. At the end of each month t, for each stock i, and for a specific forecast horizon τ, we construct the earnings prediction. The target variable of interest is the analysts' quarterly, one-year-, or two-year-ahead forecast error (i.e., the realized errors of analysts' forecasts made at month t).[24]

In our training set, we make sure that both the target variable and the predictors are known at month t. Specifically, that means the realized earnings used in constructing the target variable are known/announced before or during month t. After we fit the model, including selecting the optimal hyperparameters, we use the fitted model to generate earnings predictions at month t.

A.4 Machine Learning Methodology

We use the gradient-boosted regression tree model implemented using LightGBM (LGBM), a popular, off-the-shelf machine learning algorithm, as our main statistical forecasting model, as it provides the best outcome for predicting future earnings (Campbell et al., 2023). LGBM is a nonlinear nonparametric ensemble model which combines the predictions of many decision trees. In a process known as boosting, trees are grown sequentially to correct the prediction error from the previous iteration (Friedman, 2001). The weighted average of these individual tree models is the final predictor.

Our forecasts begin in June 1990 to allow for enough data to be available for the first training window. We train our model's hyperparameters using cross-validation, which splits the data in the training window into subsets (creating a training subset and validation subset). Then, various combinations of the hyperparameters are tested to identify the best combination. The ML model is then fit to the data using the selected hyperparameters and forecasts are made using out-of-sample data to ensure no lookahead bias.

[24]For analysts' forecasts from I/B/E/S, we use the consensus median forecasts as of the latest IBES statistical period, STATPERS, before the end of month t.

B Appendix B: Robustness Analysis Correlation

Figure A1: Correlation Matrix of Information Cost Index Components and OIV Components

This figure shows the Spearman correlations for the components of the Information Cost Index and the components of IVOL. As the Information Cost Index consists of measures that update infrequently (the Bog Index, Complexity, and Net File Size update annually and Firm Age is slow moving), the analysis is done as of the end of June in each year. The Information Cost Index is the average of the residual of the normalized rank (i.e., the rank scaled by the number of stocks in the cross section) of -LN(Firm Age), the Bog Index, and LN(Net File Size), each orthogonalized to the normalized rank of Size. For comparability, the normalized rank of the OIV related variables (OIV, OIV$_{MA36}$, Abnormal OIV, and OIV Orth.) are also orthogonalized to the normalized rank of Size. OIV Orth. is calculated by regressing OIV on demeaned OIV using a rolling 36-month window. As OIV does not vary across a given month, OIV related correlations are calculated by firm and then averaged. The sample period begins in June 1996, with the exception of OIV$_{MA36}$, Abnormal OIV, and OIV Orth. which begin in June 1998. All samples end in June 2019.

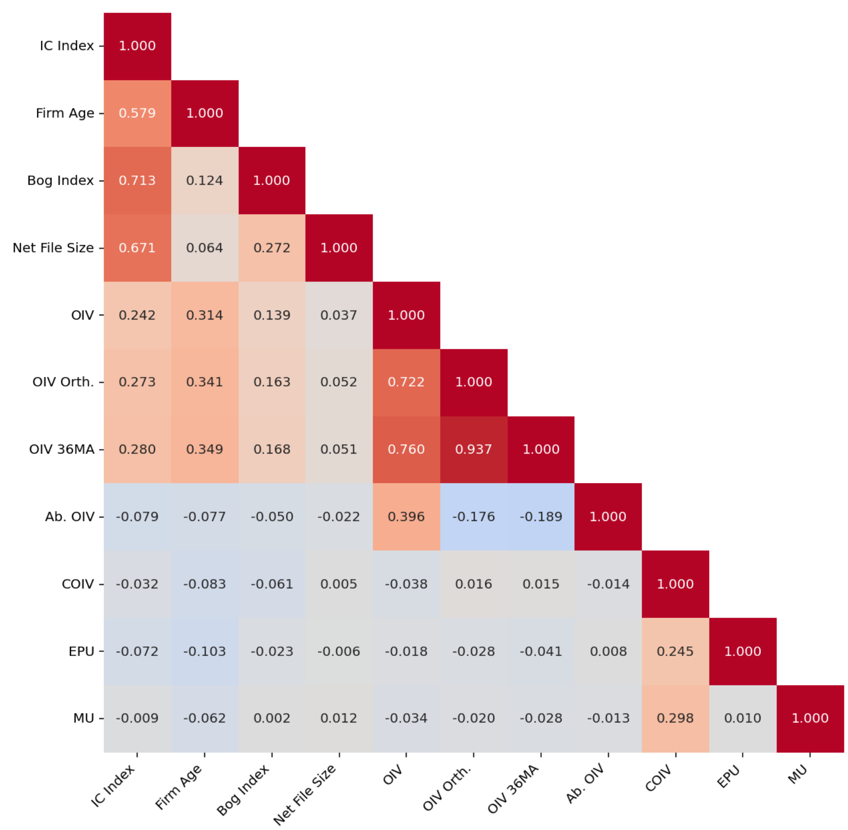

Figure A2: Return Predictability of EHB and Alternative Variables

This figure presents the Fama-French Five-Factor alphas for double-sort portfolios created by first sorting companies into terciles based on various uncertainty measures, then conditionally cross-sectionally sorting into quintiles based on EHB. Only the EHB Q1-Q5 alpha is shown for each tercile. The EHB Q1-Q5 portfolios based on $IVOL_{MA36}$ abnormal IVOL, and OIV Orth are made by first cross-sectionally sorting companies into terciles based on the specific uncertainty measure. The EHB Q1-Q5 portfolios based on COIV are made by sorting observations into terciles in the time series based on each uncertainty measure. Portfolios are value weighted and are re-balanced monthly. Q1 (Q5) contains firms with the lowest (highest) values of EHB. T1 (T3) of each uncertainty tercile contains firms with the lowest (highest) values. Whiskers denote 95% confidence bands. Standard errors of the resulting regression coefficients are computed based on Newey and West (1987) with 12 lags. The sample period is from June 1990 to December 2019, with the exception of OIV which begins in January 1996.

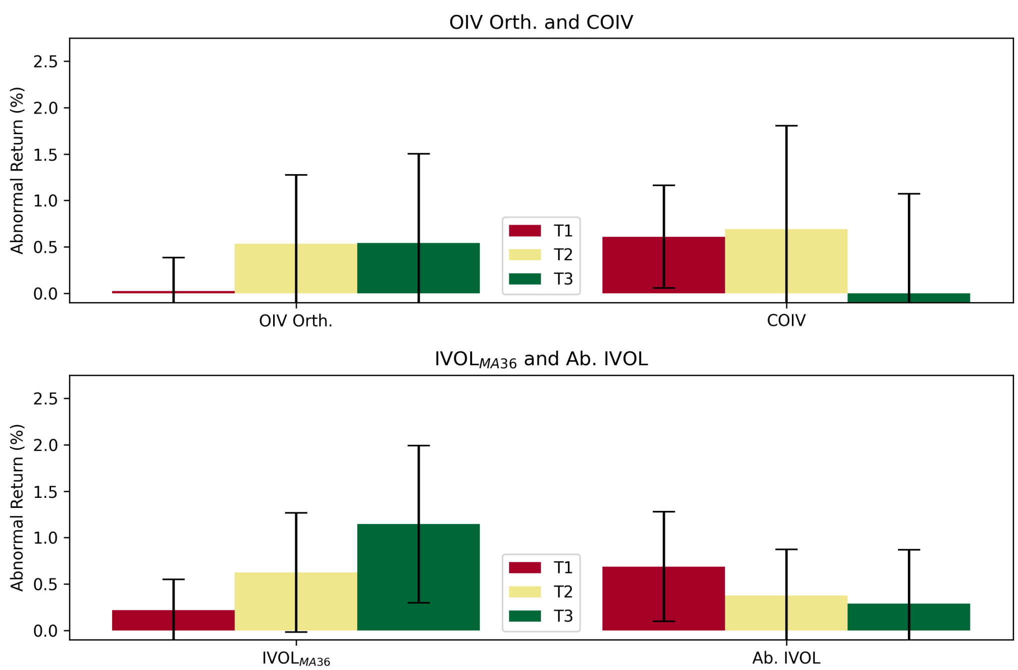

Table A3: Information Cost Index Components' Persistence

This table presents the results of panel regressions of the Bog Index, LN(Net File Size), and -LN(Firm Age) on their one-year lagged values. We use the values at the end of June in each year. Statistical significance is denoted as ***, **, and * for p<0.10, p<0.05, and p<0.01, respectively. Standard errors are clustered by firm and year. The sample period for Firm Age begins in June 1990, and the sample period for the Bog Index and Net File Size begins in June 1996. All samples end in June 2019.

	(1) -LN(Firm Age)	(2) Bog Index	(3) LN(Net File Size)
-LN(Firm Age)(t-1)	0.875***		
	(325.7)		
Bog Index(t-1)		0.916***	
		(44.5)	
LN(Net File Size) (t-1)			0.647***
			(37.9)
Cons.	-0.755***	7.651***	4.588***
	(-53.1)	(4.7)	(21.2)
Observations	45991	33667	33063

Table A4: OIV and Information Cost

This table presents the results of pooled OLS regressions of LN(OIV) on the components of the Information Cost Index (Firm Age, the Bog Index, and Net File Size) and Size. As the Information Cost Index consists of measures that update infrequently (the Bog Index and Net File Size update annually and Firm Age is slow moving), the regressions are run as of the end of June in each year. Columns 1 and 2 use -LN(Firm Age), columns 3 and 4 use the Bog Index, and columns 5 and 6 use LN(Net File Size). Statistical significance is denoted as ***, **, and * for $p<0.10$, $p<0.05$, and $p<0.01$, respectively. Standard errors are clustered at the firm and year level. Columns 1, 3, and 5 include firm fixed effects while columns 2, 4, and 6 include time fixed effects. The annual sample period for Firm Age begins in June 1990, and the sample period for the Bog Index and Net File Size begins in June 1996. All samples end in June 2019.

	-LN(Age)		Bog Index		LN(Net File Size)	
	(1)	(2)	(3)	(4)	(5)	(6)
LN(OIV)	0.234***	0.899***	-1.043	3.360***	-0.035	0.044*
	(3.1)	(13.2)	(-1.4)	(6.3)	(-0.6)	(1.7)
Size	-0.283***	-0.148***	1.865***	0.230*	0.117***	0.095***
	(-8.2)	(-8.8)	(4.9)	(2.0)	(5.1)	(12.6)
Cons.	-2.661***	-3.033***	69.139***	86.647***	11.970***	12.229***
	(-11.4)	(-26.6)	(26.1)	(101.1)	(65.7)	(233.1)
Fixed Effects	Firm	Time	Firm	Time	Firm	Time
Observations	36966	37911	34975	35905	34498	35415

Table A5: Variations in IVOL and the Return Predictability of Analysts' Conditional Biases

This table presents the Fama-French Five-Factor alphas for double-sort portfolios created by first sorting companies into terciles based on various uncertainty measures, then conditionally cross-sectionally sorting into quintiles based on EHB. The EHB Q1-Q5 portfolios based on $IVOL_{MA36}$ abnormal IVOL, and OIV Orth are made by first cross-sectionally sorting companies into terciles based on the specific uncertainty measure. The EHB Q1-Q5 portfolios based on COIV are made by sorting observations into terciles in the time series based on each uncertainty measure. Portfolios are value weighted and are re-balanced monthly. Standard errors of the resulting regression coefficients are computed based on Newey and West (1987) with 12 lags. Statistical significance is denoted as ***, **, and * for p<0.10, p<0.05, and p<0.01, respectively. The sample period is from June 1990 to December 2019, with the exception of OIV Orth, and COIV which begin in January 1996.

Panel A: IVOL_{MA36}

	EHB Q1	EHB Q2	EHB Q3	EHB Q4	EHB Q5	EHB Q1-Q5
IVOL_{MA36} T1	0.024	-0.029	-0.193***	0.006	-0.194	0.218
	(0.32)	(-0.31)	(-2.76)	(0.06)	(-1.38)	(1.29)
IVOL_{MA36} T2	0.357**	0.060	-0.123	-0.158	-0.267	0.624*
	(2.20)	(0.58)	(-1.10)	(-1.14)	(-1.15)	(1.90)
IVOL_{MA36} T3	0.700**	0.249*	0.175	-0.245	-0.445*	1.145***
	(2.27)	(1.78)	(1.21)	(-1.44)	(-1.92)	(2.64)
IVOL_{MA36} T1-T3	-0.675**	-0.279	-0.368**	0.250	0.251	-0.926**
	(-2.03)	(-1.53)	(-2.05)	(1.41)	(1.24)	(-2.39)

Panel B: Abnormal IVOL

	EHB Q1	EHB Q2	EHB Q3	EHB Q4	EHB Q5	EHB Q1-Q5
Ab. IVOL T1	0.496***	0.328**	0.023	-0.028	-0.192	0.687**
	(2.67)	(2.54)	(0.21)	(-0.25)	(-0.95)	(2.28)
Ab. IVOL T2	0.078	-0.015	-0.108	0.099	-0.298	0.376
	(0.82)	(-0.17)	(-1.53)	(0.78)	(-1.36)	(1.49)
Ab. IVOL T3	-0.069	-0.131	-0.232**	-0.051	-0.356	0.287
	(-0.58)	(-1.39)	(-2.09)	(-0.28)	(-1.37)	(0.97)
Ab. IVOL T1-T3	0.565**	0.458***	0.255	0.023	0.164	0.401
	(2.35)	(2.58)	(1.46)	(0.10)	(0.56)	(1.16)

Panel C: OIV Orth

	EHB Q1	EHB Q2	EHB Q3	EHB Q4	EHB Q5	EHB Q1-Q5
OIV Orth. T1	0.068	0.005	-0.152**	0.042	0.044	0.024
	(0.75)	(0.05)	(-2.15)	(0.39)	(0.30)	(0.13)
OIV Orth. T2	0.151	0.088	-0.088	-0.112	-0.383	0.533
	(0.77)	(0.65)	(-0.55)	(-0.65)	(-1.48)	(1.41)
OIV Orth. T3	0.311	0.161	0.085	-0.020	-0.230	0.541
	(0.97)	(0.93)	(0.50)	(-0.08)	(-0.69)	(1.10)
OIV Orth. T1-T3	-0.243	-0.156	-0.237	0.062	0.274	-0.518
	(-0.70)	(-0.69)	(-1.35)	(0.24)	(0.92)	(-1.18)

Panel D: COIV

	EHB Q1	EHB Q2	EHB Q3	EHB Q4	EHB Q5	EHB Q1-Q5
COIV T1	0.301***	0.002	-0.074	-0.078	-0.308	0.608**
	(2.70)	(0.01)	(-1.28)	(-0.49)	(-1.61)	(2.16)
COIV T2	0.231	-0.038	0.019	-0.079	-0.457	0.689
	(0.91)	(-0.55)	(0.23)	(-0.35)	(-1.38)	(1.21)
COIV T3	0.045	-0.166	0.028	0.155	0.317	-0.272
	(0.15)	(-1.25)	(0.20)	(0.59)	(0.65)	(-0.40)
COIV T1-T3	0.256	0.168	-0.102	-0.234	-0.625	0.881
	(0.80)	(1.04)	(-0.75)	(-0.80)	(-1.24)	(1.22)

Table A6: Return Predictability of EHB by Uncertainty Terciles: Mega and Non-Mega Cap

This table presents the Fama-French Five-Factor alphas for double-sort portfolios created by first cross-sectionally sorting companies into terciles based on various uncertainty measures, then conditionally cross-sectionally sorting into quintiles based on EHB. Only the EHB Q1-Q5 alpha is shown for each tercile. Panel A uses only firms in the non-mega-cap subsample, while Panel B uses only firms in the mega-cap sample. The EHB Q1-Q5 portfolios based on OIV, IVOL, $IVOL_{MA36}$, and abnormal IVOL are made by cross-sectionally sorting companies into terciles based on each uncertainty measure. The EHB Q1-Q5 portfolios based on MU and EPU are made by sorting observations into terciles in the time series based on each uncertainty measure. Portfolios are value weighted and are re-balanced monthly. Q1 (Q5) contains firms with the lowest (highest) values of EHB. T1 (T3) of each uncertainty tercile contains firms with the lowest (highest) values. Standard errors of the resulting regression coefficients are computed based on Newey and West (1987) with 12 lags. The sample period is from June 1990 to December 2019, with the exception of OIV which begins in January 1996.

Panel A: Non-Mega-Cap

	OIV Orth.	$IVOL_{MA36}$	COIV	Ab. IVOL
T1	-0.065	0.054	0.682**	0.773***
	(-0.30)	(0.29)	(2.45)	(3.26)
T2	0.236	0.602**	0.974**	0.573**
	(0.75)	(2.41)	(2.29)	(2.06)
T3	0.721*	1.198***	-0.154	0.598**
	(1.66)	(3.30)	(-0.25)	(2.03)
T1-T3	-0.786**	-1.144***	0.836	0.175
	(-2.05)	(-3.13)	(1.29)	(0.79)

Panel B: Mega-Cap

	OIV Orth.	$IVOL_{MA36}$	COIV	Ab. IVOL
T1	-0.202	-0.204	0.314	0.331
	(-0.92)	(-0.89)	(1.07)	(1.55)
T2	0.059	0.358	0.485	-0.277
	(0.28)	(1.61)	(0.78)	(-1.02)
T3	0.514	0.604	-0.534	0.344
	(1.40)	(1.64)	(-0.93)	(1.22)
T1-T3	-0.716*	-0.807*	0.848	-0.013
	(-1.66)	(-1.71)	(1.35)	(-0.05)

Table A7: Return Predictability of Announcement Return by Alternative Uncertainty Terciles: Mega and Non-Mega Cap

This table presents the Fama-French Five-Factor alphas for double-sort portfolios created by first sorting companies into terciles based on various uncertainty measures, then conditionally cross-sectionally sorting into quintiles based on Announcement Returns. Only the Announcement Return Q5-Q1 alpha is shown for each tercile. Panel A uses only firms in the non-mega-cap subsample, while Panel B uses only firms in the mega-cap sample. The Announcement Return Q5-Q1 portfolios based on OIV Orth, $Ivol_{MA36}$ and Abnormal IVOL are made by cross-sectionally sorting companies into terciles based on each uncertainty measure. The Announcement Return Q5-Q1 portfolios based on COIV are made by sorting observations into terciles in the time series based on each uncertainty measure. Portfolios are value weighted and are re-balanced monthly. Q1 (Q5) contains firms with the lowest (highest) values of Announcement Return. T1 (T3) of each uncertainty tercile contains firms with the lowest (highest) values. Standard errors of the resulting regression coefficients are computed based on Newey and West (1987) with 12 lags. The sample period is from June 1990 to December 2019, with the exception of OIV Orth and COIV which begin in January 1996.

Panel A: Non-Mega-Cap

	OIV Orth.	$IVOL_{MA36}$	COIV	Ab. IVOL
T1	0.153	0.299***	0.482***	0.669***
	(1.31)	(3.42)	(5.19)	(4.95)
T2	0.213	0.353***	0.483**	0.396***
	(1.43)	(2.62)	(2.32)	(2.76)
T3	0.298	0.804***	0.349	0.382**
	(1.10)	(4.00)	(1.48)	(2.39)
T1-T3	-0.145	-0.506***	0.133	0.288
	(-0.58)	(-2.59)	(0.46)	(1.62)

Panel B: Mega-Cap

	OIV Orth.	$IVOL_{MA36}$	COIV	Ab. IVOL
T1	-0.230	0.019	0.222	0.225
	(-1.64)	(0.12)	(1.41)	(1.37)
T2	0.096	0.176	0.470**	0.297
	(0.54)	(0.90)	(2.04)	(1.38)
T3	0.854**	0.786***	0.264	0.439**
	(2.40)	(2.61)	(0.63)	(2.21)
T1-T3	-1.083**	-0.767**	-0.042	-0.215
	(-2.52)	(-2.15)	(-0.10)	(-0.96)

Table A8: Return Predictability of Analysts' Revisions by Alternative Uncertainty Terciles: Mega and Non-Mega Cap

This table presents the Fama-French Five-Factor alphas for double-sort portfolios created by first sorting companies into terciles based on various uncertainty measures, then conditionally cross-sectionally sorting into quintiles based on Analysts' Revisions. Only the Analysts' Revisions Q5-Q1 alpha is shown for each tercile. Panel A uses only firms in the non-mega-cap subsample, while Panel B uses only firms in the mega-cap sample. The Analysts' Revisions Q5-Q1 portfolios based on OIV Orth, Ivol_{MA36} and Abnormal IVOL are made by cross-sectionally sorting companies into terciles based on each uncertainty measure. The Analysts' Revisions Q5-Q1 portfolios based on COIV are made by sorting observations into terciles in the time series based on each uncertainty measure. Portfolios are value weighted and are re-balanced monthly. Q1 (Q5) contains firms with the lowest (highest) values of Analysts' Revisions. T1 (T3) of each uncertainty tercile contains firms with the lowest (highest) values. Standard errors of the resulting regression coefficients are computed based on Newey and West (1987) with 12 lags. The sample period is from June 1990 to December 2019, with the exception of OIV Orth and COIV which begin in January 1996.

Panel A: Non-Mega-Cap

	OIV Orth.	IVOL_{MA36}	COIV	Ab. IVOL
T1	-0.104	0.139	0.651***	0.396**
	(-0.51)	(0.84)	(4.05)	(2.48)
T2	-0.029	0.275	0.832**	0.105
	(-0.09)	(1.09)	(2.15)	(0.45)
T3	-0.059	0.271	-0.892	0.150
	(-0.17)	(0.91)	(-1.62)	(0.53)
T1-T3	-0.045	-0.132	1.543***	0.246
	(-0.15)	(-0.51)	(2.64)	(1.01)

Panel B: Mega-Cap

	OIV Orth.	IVOL_{MA36}	COIV	Ab. IVOL
T1	-0.114	0.060	0.417*	0.469***
	(-0.57)	(0.28)	(1.87)	(2.59)
T2	0.270	0.354	0.768*	0.081
	(0.86)	(1.25)	(1.93)	(0.39)
T3	0.921***	0.877***	0.019	0.484*
	(3.49)	(3.84)	(0.04)	(1.86)
T1-T3	-1.035***	-0.817**	0.398	-0.015
	(-2.85)	(-2.38)	(0.69)	(-0.05)

Table A9: Testing Alternative Theories using OIV

This table presents the results of monthly regressions of |EHB|, EDGAR Downloads, or Effective Spread on the measures of uncertainty. The |EHB| and Effective Spread analysis use pooled OLS regressions. As EDGAR Downloads are a count measure, a pseudo-Poisson regression is used instead of a pooled OLS regression. Columns 1, 3, and 5 include only Size as a control, and columns 2, 4, and 6 add Firm Age and an indicator equal to one in earnings-announcement months. Standard errors are clustered at the firm and month level. Statistical significance is denoted as ***, **, and * for p<0.10, p<0.05, and p<0.01, respectively. The sample period for |EHB| is from January 1996 to December 2019 as OIV does not have observations prior to this. The sample for EDGAR Downloads begins in April 2003 and ends in December 2017. The sample for Effective Spread begins in September 2003 and ends in December 2019.

	|EHB|		EDGAR Downloads		Eff. Spread	
Size	-0.097***	-0.116***	0.623***	0.635***	-2.720***	-2.667***
	(-8.6)	(-9.8)	(8.9)	(8.1)	(-18.1)	(-16.9)
MU	1.547***	1.408***	-2.582***	-2.528***	5.920***	6.200***
	(4.5)	(4.3)	(-5.6)	(-6.0)	(2.7)	(2.8)
LN(OIV)	0.872***	0.969***	0.693***	0.666***	4.968***	4.789***
	(19.0)	(19.1)	(8.1)	(11.3)	(9.5)	(9.3)
LN(Age)		0.122***		-0.068		-0.323**
		(9.7)		(-1.0)		(-2.0)
Earn. Annc.		0.093***		0.145***		0.173
		(7.6)		(3.3)		(0.7)
Observations	447100	447100	259430	259430	300314	300314

Table A10: Testing Alternative Theories using AIA

This table presents the results of probit daily regressions of AIA on the measures of uncertainty. AIA is an indicator equal to 1 when Bloomberg News Heat-Daily Max Readership Measure is 3–4 and 0 otherwise. Columns 1 and 3 include Size as a control and columns 2 and 4 add Firm Age and an indicator equal to one in earnings-announcement months. All dependent variables are measured at the monthly level. Standard errors are clustered at the firm and day level. Statistical significance is denoted as ***, **, and * for p<0.10, p<0.05, and p<0.01, respectively. The sample period is from March 2010 to December 2019.

	(1)	(2)	(3)	(4)
Size	0.284***	0.286***	0.279***	0.289***
	(54.6)	(53.5)	(50.7)	(51.5)
EPU	-0.001***	-0.000***		
	(-3.7)	(-2.9)		
LN(IVOL)	0.388***	0.386***		
	(26.5)	(24.9)		
LN(Age)		-0.004		-0.016**
		(-0.6)		(-2.5)
Earn. Annc.		0.213***		0.249***
		(21.8)		(25.3)
MU			-1.866***	-1.856***
			(-10.6)	(-10.5)
LN(OIV)			0.372***	0.406***
			(22.0)	(23.0)
Observations	4006685	4006685	3920270	3920270

C Appendix C: Derivations, Proofs, and Extension of the Model

C.1 Derivations and Proofs

Proof of Lemma 1

Proof. The demand function can be written as

$$
\begin{aligned}
\tilde{q}_j^s &= \frac{1}{\gamma}\hat{\Sigma}_j^{-1}\left[E_j(\tilde{f}) - \tilde{p}\right] \\
&= \frac{1}{\gamma}\hat{\Sigma}_j^{-1}\left[\Gamma^{-1}\mu + E_j(z) - \tilde{p}\right] \\
&= \frac{1}{\gamma}\left[\hat{\Sigma}_j^{-1}\left(\Gamma^{-1}\mu - \tilde{p}\right) + \Sigma_{\eta j}^{-1}(z + (\mathbf{I} - \mathbf{b}_j)B)\right]
\end{aligned}
$$

$\mathbf{I}$ is an N-dimension identical matrix, $\mathbf{b}_j$ is a diagonal matrix with the i-th diagonal element being b_{ij}. The aggregated demand is

$$
\int \tilde{q}_j dj = \frac{1}{\gamma}\left[\bar{\Sigma}^{-1}\left(\Gamma^{-1}\mu - \tilde{p}\right) + \int \hat{\Sigma}_{\eta j}^{-1}\left(z + (1 - b_j)B\right) dj\right] \tag{A1}
$$

where $\bar{\Sigma}^{-1} = \int \hat{\Sigma}_j^{-1} dj$ is the aggregate posterior precision matrix.

Since there is no heterogeneity among skilled investors, we study a symmetric equilibrium where every investor will choose the same level of de-biasing for a given stock, which we denoted by a diagonal matrix $\mathbf{b}$. The private signal variance will then be the same for each skilled investor, which we denote by $\Sigma_\eta = (\mathbf{I} - \mathbf{b})\Sigma(\mathbf{I} - \mathbf{b})'$. The aggregate posterior precision is $\bar{\Sigma}^{-1} = \Sigma^{-1} + \Sigma_\eta^{-1}$.

Applying the market clear condition 3 and matching the coefficients for the intercept and different shocks, we get the following equations,

$$
\bar{\Sigma}^{-1}(\Gamma^{-1}\mu - A_0) = \gamma x \tag{A2}
$$

$$
-\bar{\Sigma}^{-1}A_z + \Sigma_\eta^{-1} = 0 \tag{A3}
$$

$$
-\bar{\Sigma}^{-1}A_B + \Sigma_\eta^{-1}(\mathbf{I} - \mathbf{b}) = 0 \tag{A4}
$$

From the above equations we get $A_0 = \Gamma^{-1}\mu - \gamma\bar{\Sigma}x$, $A_z = \bar{\Sigma}\Sigma_\eta^{-1}$, and $A_B = \bar{\Sigma}\Sigma_\eta^{-1}(\mathbf{I} - \mathbf{b})$. □

Proof of Corollary 1

Proof. Define the excess return of a stock as $r^e = f - \Gamma\tilde{p}$. Then

$$
\begin{aligned}
r^e &= f - \Gamma\tilde{p} \\
&= \gamma\Gamma\bar{\Sigma}x + \Gamma(I - A_z)z - \Gamma A_z(I - b)B \\
&= \Gamma\left(\gamma\bar{\Sigma}x + \bar{\Sigma}\Sigma^{-1}z - \bar{\Sigma}\Sigma_\eta^{-1}(I - b)B\right)
\end{aligned}
$$

Given the matrix structure of Γ, each stock's excess return can be written as a part that loads on the market excess return, and a part that contingent on the shocks. □

Proof of Corollary 2

Proof. It's obvious that $\zeta_i^B > 0$. It remains to prove that $\frac{d\zeta_i^B}{db_i} < 0$. Note that

$$\zeta_i^B = \frac{\sigma_{\eta i}^{-1}}{\sigma_i^{-1} + \sigma_{\eta i}^{-1}}(1-b_i) = \frac{1}{\frac{\sigma_i^B}{\sigma_i}(1-b_i) + \frac{1}{1-b_i}}$$

The equation holds because $\sigma_{\eta i} = \sigma_i^B(1-b_i)^2$. The denominator is increasing in b_i since the derivative $-\frac{\sigma_i^B}{\sigma_i} + \frac{1}{(1-b_i)^2} \geq 0$ due to $\frac{\sigma_i^B}{\sigma_i} = \rho < 1$. Therefore $\frac{d\zeta_i^B}{db_i} < 0$: higher de-biasing decreases the return predictability of analyst forecast bias. □

Proof of Lemma 2

Proof. Put the expression of the demand function $\tilde{q}_j$ to U_{0j} in Equation 5,

$$\begin{aligned}
U_{0j} &= W_0 + E_0\left[\tilde{q}_j' E_j\left(\tilde{f} - \tilde{p}\right) - \frac{\gamma}{2}\tilde{q}_j' V_j\left(\tilde{f} - \tilde{p}\right)\tilde{q}_j\right] - \sum_{i=1}^{n} c_{ij} \\
&= W_0 + \frac{1}{\gamma}E_0\left[\left(E_j(\tilde{f}) - \tilde{p}\right)'\hat{\Sigma}_j^{-1}\left(E_j(\tilde{f}) - \tilde{p}\right) - \frac{1}{2}\left(E_j(\tilde{f}) - \tilde{p}\right)'\hat{\Sigma}_j^{-1}\hat{\Sigma}_j\hat{\Sigma}_j^{-1}\left(E_j(\tilde{f}) - \tilde{p}\right)\right] - \sum_{i=1}^{n} c_{ij} \\
&= W_0 + \frac{1}{2\gamma}E_0\left[\left(E_j(\tilde{f}) - \tilde{p}\right)'\hat{\Sigma}_j^{-1}\left(E_j(\tilde{f}) - \tilde{p}\right)]\right] - \sum_{i=1}^{n} c_{ij}
\end{aligned}$$

Note that $E_j(\tilde{f}) - \tilde{p}$ is normally distributed. Thus U_{0j} is an expectation of a non-central χ^2-distributed random variable. This equals

$$\begin{aligned}
U_{0j} &= W_0 + \frac{1}{2\gamma}\left[\text{Trace}\left[\hat{\Sigma}_j^{-1}V_0\left(E_j(\tilde{f}) - \tilde{p}\right)\right] + E_0\left(E_j(\tilde{f}) - \tilde{p}\right)'\hat{\Sigma}_j^{-1}E_0\left(E_j(\tilde{f}) - \tilde{p}\right)\right] - \sum_{i=1}^{n} c_{ij} \\
&= W_0 + \frac{1}{2\gamma}\left[\text{Trace}\left[\hat{\Sigma}_j^{-1}V_0\left(\tilde{f} - \tilde{p}\right) - I\right] + \gamma^2 x'\bar{\Sigma}'\hat{\Sigma}_j^{-1}\bar{\Sigma}x\right] - \sum_{i=1}^{n} c_{ij}
\end{aligned}$$

where Trace$(\cdot)$ is the trace of a matrix. The second equality applies the Law of Total Variance $V_0\left(E_j\left(\tilde{f}\right) - \tilde{p}\right) = V_0\left(\tilde{f} - \tilde{p}\right) - E_0\left(V_j\left(\tilde{f}_j - \tilde{p}\right)\right) = V_0\left(\tilde{f} - \tilde{p}\right) - \hat{\Sigma}_j$ Note that

$$\begin{aligned}
V &\equiv V_0\left(\tilde{f} - \tilde{p}\right) \\
&= (I - A_z)\Sigma(I - A_z)' + A_B\Sigma A_B' \\
&= \bar{\Sigma}\left[\Sigma^{-1} + \Sigma_\eta^{-1}(I-b)\Sigma(I-b)\Sigma_\eta^{-1}\right]\bar{\Sigma}' \\
&= \bar{\Sigma}\left[\Sigma^{-1} + \Sigma_\eta^{-1}\right]\bar{\Sigma}' \\
&= \bar{\Sigma}
\end{aligned}$$

The ith diagonal element of V is then the posterior variance $\bar{\sigma}_i$. Given the diagonal nature of the problem, the ex-ante expected utility is given by

$$\begin{aligned}
U_{0j} =& W_0 + \frac{1}{2\gamma}\sum_{i=1}^{n}\left(\tau_i + \tau_{ij}^{\eta}\right)\bar{\sigma}_i - \frac{n}{2\gamma} + \frac{1}{2\gamma}\sum_{i=1}^{n}\gamma^2\bar{\sigma}_i^2 x_i^2\left(\tau_i + \tau_{ij}^{\eta}\right) - \sum_{i=1}^{n} c_{ij} \\
=& constant + \frac{1}{2\gamma}\sum_{i=1}^{n}\tau_{ij}^{\eta}\left(\bar{\sigma}_i + \gamma^2\bar{\sigma}_i^2 x_i^2\right) - \sum_{i=1}^{n} c_{ij} \\
=& constant + \sum_{i=1}^{n}\lambda_i\frac{\tau_{ij}^{\eta}}{\tau_i^B} - \sum_{i=1}^{n}\frac{\kappa_i}{2}\left(\frac{\tau_{ij}^{\eta}}{\tau_i^B} - 1\right)^2
\end{aligned}$$

where $\lambda_i = \frac{1}{2\gamma\sigma_i^B}\left(\bar{\sigma}_i + \gamma^2\bar{\sigma}_i^2 x_i^2\right)$.

The time-0 expected utility is a *quadratic function* on the relative precision of the de-biased signal $\frac{\tau_{ij}^{\eta}}{\tau_i^B}$. The marginal benefit of increasing relative signal precision is a constant given by λ_i. Given the quadratic information cost in Equation 4. The optimal learning decision is given by $\tau_{ij}^{\eta} = \tau_i^B\left(1 + \frac{\lambda_i}{\kappa_i}\right)$. Note that $\frac{\tau_{ij}^{\eta}}{\tau_i^B} = \frac{1}{(1-b_{ij})^2}$, the optimal level of de-biasing is $b_{ij} = 1 - \sqrt{\frac{\kappa_i}{\lambda_i + \kappa_i}}$. □

Proof of Corollary 3

Proof. Note that $\lambda_i = \frac{1}{2\gamma\sigma_i^B}\left(\bar{\sigma}_i + \bar{\sigma}_i^2\gamma^2 x_i^2\right)$ and the posterior $\bar{\sigma}_i = \frac{1}{\tau_i^{\eta} + \tau_i} = \frac{\sigma_i^B}{\theta_i + \rho}$. Therefore, the marginal benefit λ_i can be expressed as a function on θ_i as follows

$$\lambda_i = \frac{1}{2\gamma}\left(\frac{1}{\theta_i + \rho} + \frac{\sigma_i^B}{(\theta_i + \rho)^2}\gamma^2 x_i^2\right)$$

Clearly, $\frac{d\lambda_i}{d\theta_i} < 0$. Given that $\theta_i = \frac{1}{(1-b_i)^2}$ is positively related to the de-biasing level b_i, we get $\frac{d\lambda_i}{db_i} < 0$. □

Proof of Proposition 1

Proof. We first prove that the de-biasing level is decreasing in the intrinsic volatility, i.e., $\frac{db_i}{d\sigma_i^F} < 0$. This is equivalent to show that the relative signal precision θ_i is decreasing in σ_i^F, since there is a positive monotonic relationship between θ_i and b_i given by $\theta_i = \frac{1}{(1-b_i)^2}$. Note that the equilibrium is determined by solving the fixed-point problem:

$$f(\theta_i) = \kappa_i(\theta_i - 1) - \lambda_i = 0$$

Applying the Implicit Function Theorem,

$$\frac{d\theta_i}{d\sigma_i^F} = -\frac{\partial f/\partial\sigma_i^F}{\partial f/\partial\theta_i} = -\frac{\psi(\theta_i - 1) - \frac{\partial\lambda_i}{\partial\sigma_i^F}}{\kappa_i - \frac{\partial\lambda_i}{\partial\theta_i}} \tag{A5}$$

According to the proof of Corollary 3, $\frac{\partial\lambda_i}{\partial\theta_i} < 0$. Therefore the denominator in Equation A5 is positive.

Examining the numerator in Equation A5, note that

$$\frac{\partial\lambda_i}{\partial\sigma_i^F} = \frac{1}{2}\frac{\rho}{(\theta_i+\rho)^2}\gamma x_i^2$$

The equation holds because $\sigma_i^B = \rho\sigma_i = \rho(\sigma_i^F + \sigma_i^S)$. In addition, $\psi(\theta_i - 1) = \frac{\kappa_i(\theta_i-1)}{\sigma_i^F} = \frac{\lambda_i}{\sigma_i^F}$. Thus the numerator can be written as

$$\frac{\lambda_i}{\sigma_i^F} - \frac{\partial\lambda_i}{\partial\sigma_i^F} = \frac{1}{2\gamma\sigma_i^F}\frac{1}{\theta_i+\rho} + \frac{1}{2}\frac{\rho}{(\theta_i+\rho)^2}\frac{\sigma_i^S}{\sigma_i^F}\gamma x_i^2 > 0$$

Thus the numerator in Equation A5 is positive.

Overall, we get $\frac{d\theta_i}{d\sigma_i^F} < 0$ and therefore $\frac{db_i}{d\sigma_i^F} < 0$, i.e., higher intrinsic uncertainty decreases de-biasing level. According to Corollary 2, return predictability is decreasing with the de-biasing level unconditionally, $\frac{d\zeta_i^B}{db_i} < 0$. Thus $\frac{d\zeta_i^B}{d\sigma_i^F} > 0$. Higher intrinsic uncertainty increases return predictability of the analyst forecast bias.

□

Proof of Proposition 2

Proof. The proof follows the same strategy as that of Proposition 1. We first prove that $\frac{d\theta_i}{d\sigma_i^S} > 0$. Applying the Implicit Function Theorem,

$$\frac{d\theta_i}{d\sigma_i^S} = -\frac{\partial f/\partial\sigma_i^S}{\partial f/\partial\theta_i} = \frac{\frac{\partial\lambda_i}{\partial\sigma_i^S}}{\kappa_i - \frac{\partial\lambda_i}{\partial\theta_i}} \tag{A6}$$

We have shown in the proof of Proposition 1 that the denominator in equation (A6) is positive. Given $\lambda_i = \frac{1}{2\gamma}\left(\frac{1}{\theta_i+\rho} + \frac{\sigma_i^B}{(\theta_i+\rho)^2}\gamma^2 x_i^2\right)$ and $\sigma_i^B = \rho(\sigma_i^F + \sigma_i^S)$, then

$$\frac{\partial\lambda_i}{\partial\sigma_i^S} = \frac{1}{2}\frac{\rho}{(\theta_i+\rho)^2}\gamma x_i^2 > 0$$

Therefore the numerator in equation (A6) is positive. Overall we show that $\frac{d\theta_i}{d\sigma_i^S} > 0$ and therefore $\frac{db_i}{d\sigma_i^S} > 0$. That is, higher temporal uncertainty encourages de-biasing. According to Corollary 2, it is immediate that $\frac{d\zeta_i^B}{d\sigma_i^S} < 0$, higher temporal uncertainty decreases return predictability. □

Proof of Proposition 3

Proof. We first prove that more de-biasing increases the price efficiency, that is, $\frac{dA_{z,i}}{db_i} < 0$. Note that

$$A_{z,i} = \frac{\tau_i^\eta}{\tau_i^\eta + \tau_i} = \frac{1}{1 + \rho(1 - b_i)^2}$$

The equation holds because $\sigma_i^\eta = \sigma_i^B(1 - b_i)^2$ and $\sigma_i^B = \rho\sigma_i$. Therefore $\frac{dA_{z,i}}{db_i} > 0$, that is, higher de-biasing leads to more precise signals and stronger price sensitivity to fundamental shocks (higher price efficiency).

According to the proof in Proposition 1 and Proposition 2, higher σ_i^F (σ_i^S) decreases (increases) de-biasing level and therefore decreases (increases) price efficiency. Note that $\zeta_i^z = 1 - A_{z,i}$, the opposite holds for the fundamental-based return anomaly.

□

C.2 Extension

C.2.1 Generalized Model Setup

In this section, we present the fully specified model where we have both informed and uninformed investors and supply noise, consistent with the standard information choice model in the literature. In the fully-specified model, we adopt the same asset payoff structure, preferences, and the de-biasing process and cost functions in the simplified model. In addition, we maintain two key structures. First, the variance of analyst's forecast bias is proportional to the prior variance, $\sigma_i^B = \rho\sigma_i$ where $\rho < 1$; Second, the information cost parameter is proportional to the intrinsic uncertainty $\kappa_i = \psi\sigma_i^F$. The generalization comes from the following two aspects.

First, we assume stochastic supply of factors, denoted by $\bar{x}_i + x_i$ for factor i, where $\bar{x}$ is a vector of the fixed supply and $x \sim N(0, \Sigma_x)$ being the noisy supply vector with a diagonal variance-covariance matrix given by Σ_x. The supply for asset is then $\Gamma^{-1}(\bar{x} + x)$. As in the literature, the random supply is to prevent price from fully revealing the information of informed investors. Therefore each investor will use price as a public signal to update belief on the distribution of the fundamentals.

Second, we assume a fraction χ of investors are skilled, that is, they can reduce the variance of analyst's forecast bias through information acquisition. A $1 - \chi$ fraction of investors are unskilled, their only signal is the public price signal obtained with no cost. Following this strand of literature, we conjecture and prove a linear functional form of the price, so that the price is a linear unbiased signal on the fundamental shock z, i.e., $\eta_p = z + \epsilon_p$. The signal noise is distributed as $N(0, \Sigma_p)$, where Σ_p is the diagonal variance matrix determined in equilibrium.

Posteriors Based on the private and the public signals, an investor updates her beliefs about the factors by forming a Bayesian posterior with mean and variance. We present the posteriors for skilled and unskilled investors separately. Given the homogeneity of skilled investors, equilibrium will besymmetric, i.e., every skilled investor will choose the same level of de-biasing for a given factor. In this case, all skilled investors get the same private signal, and the price signal is a noisier version of this signal (as shown later, $\varepsilon_{pi} = \varepsilon_i - \gamma\bar{\sigma}_{\eta i} x_i$). This is a key difference between our model with traditional information choice model, where skilled investors' signals are independent. Therefore, a skilled (unskilled) investor updates her belief based on the private (public price) signal. See below,

$$\text{For skilled investors: } \hat{\mu}_j \equiv E_j\left[z|\eta_j, \eta_p\right] = E_j\left[z|\eta_j\right] = \hat{\Sigma}_j \Sigma_{\eta j}^{-1} \eta_j, \quad \hat{\Sigma}_j^{-1} = \Sigma^{-1} + \Sigma_{\eta j}^{-1}$$

$$\text{For unskilled investors: } \hat{\mu}_j \equiv E_j\left[z|\eta_p\right] = \hat{\Sigma}_j \Sigma_p^{-1} \eta_p, \quad \hat{\Sigma}_j^{-1} = \Sigma^{-1} + \Sigma_p^{-1}$$

where η_j is a vector of signals that a skilled investor j obtains through de-biasing, the i-th element, $\eta_{ij} = z_i + (1 - b_{ij})B_i$ where b_{ij} is the de-biasing levels and B_i is analyst's forecast bias for factor i. $\hat{\Sigma}_j$ is investor j's posterior variance on the factors z. From time-0 perspective, $\hat{\mu}$ is normally distributed with zero mean and variance-covariance matrix $V_0\left[\hat{\mu}_j\right] = \Sigma - \hat{\Sigma}_j$ according to the law of total variance.

In the symmetric equilibrium, the aggregate posterior precision is $\bar{\Sigma}^{-1} = \int \hat{\Sigma}_j^{-1} dj = \Sigma^{-1} + \bar{\Sigma}_\eta^{-1} + \bar{\Sigma}_p^{-1}$, where $\bar{\Sigma}_\eta^{-1} = \int \Sigma_{\eta j}^{-1} dj = \chi \Sigma_\eta^{s^{-1}}$ where $\Sigma_\eta^{s^{-1}}$ is the common signal precision for all skilled investors, and $\bar{\Sigma}_p^{-1} = (1-\chi)\Sigma_p^{-1}$ is the aggregated price signal precision.

C.2.2 Solutions

We solve the model backward. First, we solve the portfolio optimization problem at $t = 1$, taking the information acquisition and posterior beliefs as given. In this step, we can also derive the equilibrium price. In the second step, we derive the optimal information acquisition problem and produce propositions about the relation between uncertainty and information acquisition.

Portfolio allocation The optimization problem is given by

$$\begin{aligned} \max_{\tilde{q}_j} \quad & U_{1j} = E_j\left[W_j\right] - \tfrac{\gamma}{2} \mathrm{V}_j\left[W_j\right] \\ \text{s.t.} \quad & W_j = W_0 + \tilde{q}_j{}'(\tilde{f} - \tilde{p}) \end{aligned}$$

which gives the solution

$$\tilde{q}_j = \frac{1}{\gamma} \hat{\Sigma}_j^{-1} \left[E_j(\tilde{f}) - \tilde{p} \right] \tag{A7}$$

Then we plugin this demand function to the market clear condition, $\int \tilde{q}_j dj = \overline{x} + x$, and obtains the following Lemma.

Lemma A1. *The equilibrium price of the factors is*

$$\tilde{p} = A_0 + A_z z + A_B B + A_x x$$

where

$$\begin{aligned} A_0 &= \Gamma^{-1}\mu - \gamma \bar{\Sigma} \overline{x} \\ A_z &= I - \bar{\Sigma}\Sigma^{-1} \\ A_B &= A_z(I - b^s) \\ A_x &= -\gamma \bar{\Sigma} \left(I + \bar{\Sigma}_p^{-1} \bar{\Sigma}_\eta \right) \end{aligned}$$

b^s*,* $\bar{\Sigma}$*,* $\bar{\Sigma}_\eta$*, and* $\bar{\Sigma}_p$ *are given below in the proof.*

Proof. For skilled investors, the demand function can be written as

$$\begin{aligned}
\tilde{q}_j^s &= \frac{1}{\gamma}\hat{\Sigma}_j^{-1}\left[E_j(\tilde{f}) - \tilde{p}\right] \\
&= \frac{1}{\gamma}\hat{\Sigma}_j^{-1}\left[\Gamma^{-1}\mu + E_j(z) - \tilde{p}\right] \\
&= \frac{1}{\gamma}\left[\hat{\Sigma}_j^{-1}\left(\Gamma^{-1}\mu - \tilde{p}\right) + \Sigma_{\eta j}^{-1}(z + (1-b_j)B)\right]
\end{aligned}$$

b_j is a diagonal matrix with the i-th diagonal element being b_{ij}. For unskilled investors, they depends on the public price signal. With the guessed form $\tilde{p} = A_0 + A_z z + A_B B + A_x x$, the price signal is given by $\eta_p = A_z^{-1}(\tilde{p} - A_0) = z + A_z^{-1}A_B B + A_z^{-1}A_x x$. Thus, demand for unskilled investor is given by

$$\tilde{q}_j^u = \frac{1}{\gamma}\left[\hat{\Sigma}_j^{-1}\left(\Gamma^{-1}\mu - \tilde{p}\right) + \Sigma_p^{-1}(z + A_z^{-1}A_B B + A_z^{-1}A_x x))\right]$$

The price signal variance is $\Sigma_p = A_z^{-1}A_B\Sigma_B A_B' A_z'^{-1} + A_z^{-1}A_x\Sigma_x A_x' A_z'^{-1}$. The aggregated demand is

$$\int \tilde{q}_j dj = \frac{1}{\gamma}\left[\bar{\Sigma}^{-1}\left(\Gamma^{-1}\mu - \tilde{p}\right) + \int \hat{\Sigma}_{\eta j}^{-1}\left(z + (1-b_j)B\right)dj + (1-\chi)\Sigma_p^{-1}(z + A_z^{-1}A_B B + A_z^{-1}A_x x))\right] \tag{A8}$$

where $\bar{\Sigma}^{-1} = \int \hat{\Sigma}_j^{-1} dj$ is the aggregate posterior precision matrix.

Since there is no heterogeneity among skilled investors, we study a symmetric equilibrium where every skilled investor will choose the same level of de-biasing for a given stock, which we denoted by a diagonal matrix b^s. The private signal variance will then be the same for each skilled investor, which we denote by $\Sigma_\eta^s = (I - b^s)\Sigma_B(I - b^s)'$. Given that unskilled investors do not de-bias analyst forecasts, we can get the aggregate precision of private signal as $\bar{\Sigma}_\eta^{-1} \equiv \int \hat{\Sigma}_{\eta j}^{-1} dj = \chi \Sigma_\eta^{s^{-1}}$. Denote the aggregate price signal precision as $\bar{\Sigma}_p^{-1} = (1-\chi)\Sigma_p^{-1}$, then the aggregate posterior precision is $\bar{\Sigma}^{-1} = \Sigma^{-1} + \bar{\Sigma}_\eta^{-1} + \bar{\Sigma}_p^{-1}$.

Applying the market clear condition where aggregated demand equals supply, $\int \tilde{q}_j dj = \bar{x} + x$, and matching the coefficients for the intercept and different shocks, we get the following equations,

$$\bar{\Sigma}^{-1}(\Gamma^{-1}\mu - A_0) = \gamma\bar{x} \tag{A9}$$

$$-\bar{\Sigma}^{-1}A_z + \bar{\Sigma}_\eta^{-1} + \bar{\Sigma}_p^{-1} = 0 \tag{A10}$$

$$-\bar{\Sigma}^{-1}A_B + \bar{\Sigma}_\eta^{-1}(I - b^s) + \bar{\Sigma}_p^{-1}A_z^{-1}A_B = 0 \tag{A11}$$

$$-\bar{\Sigma}^{-1}A_x + \bar{\Sigma}_p^{-1}A_z^{-1}A_x = \gamma I_n \tag{A12}$$

We solve the above equations as follows. From Equation A9 and A10 we get $A_0 = \Gamma^{-1}\mu - \gamma\bar{\Sigma}\bar{x}$ and $A_z = \bar{\Sigma}(\bar{\Sigma}_\eta^{-1} + \bar{\Sigma}_p^{-1}) = I - \bar{\Sigma}\Sigma^{-1}$. Multiplying Equation A10 by $A_z^{-1}A_B$ on the right and subtracting by Equation A11, we get $\bar{\Sigma}_\eta^{-1}A_z^{-1}A_B = \bar{\Sigma}_\eta^{-1}(I - b^s)$. Since the posterior precision matrix is non-singular, we have $A_B = A_z(I - b^s)$. Lastly, multiplying Equation A10 by $A_z^{-1}A_x$ on the right and subtracting by Equation A12, we get $\bar{\Sigma}_\eta^{-1}A_z^{-1}A_x = -\gamma I_n$ or $A_x = -\gamma A_z\bar{\Sigma}_\eta = -\gamma\bar{\Sigma}(I + \bar{\Sigma}_p^{-1}\bar{\Sigma}_\eta)$.

Lastly, we derive the expression for the price signal variance Σ_p,

$$\begin{aligned}
\Sigma_p &= A_z^{-1}A_B\Sigma_B A_B' A_z'^{-1} + A_z^{-1}A_x\Sigma_x A_x' A_z'^{-1} \\
&= (I - b^s)\Sigma_B(I - b^s)' + \gamma^2\bar{\Sigma}_\eta\Sigma_x\bar{\Sigma}_\eta' \\
&= \Sigma_\eta^s + \gamma^2\bar{\Sigma}_\eta\Sigma_x\bar{\Sigma}_\eta' \\
&= \chi\bar{\Sigma}_\eta + \gamma^2\bar{\Sigma}_\eta\Sigma_x\bar{\Sigma}_\eta' \qquad \text{(A13)}
\end{aligned}$$

□

Lemma A1 shows that the equilibrium price is a linear function on the fundamental shocks z, analysts' bias B and the noise in the supply x. As in the simplified model, the price loading on analysts' bias, A_B, is proportional to that on the fundamental shock, A_z,, with the proportion being $I - b^s$, i.e., the investors' de-biasing level. When investors fully de-bias analyst forecasts ($b^s = 1$), the price is not related to the bias. In contrast, if investors do not de-bias analyst forecasts at all ($b^s = 1$), the price respond to the bias as much as it would to the fundamental shock.

Lemma A1 also tells us the excess returns of each stock, defined by $r^e = f - \Gamma\tilde{p}$. Specifically, we derive the following corollary.

Corollary A1. *The excess return of stock i is*

$$r_i^e = \gamma\bar{\sigma}_i\bar{x}_i + \beta_i r_n^e + \zeta_i^z z_i - \zeta_i^B B_i + \zeta_i^x x_i, \quad \forall i = 1, \cdots, n-1 \qquad \text{(A14)}$$

$$r_n^e = \gamma\bar{\sigma}_n\bar{x}_n + \zeta_n^z z_n - \zeta_n^B B_n + \zeta_n^x x_n$$

where

$$\begin{aligned}
\zeta_i^z &= \tfrac{\bar{\sigma}_i}{\sigma_i} \\
\zeta_i^B &= \left(1 - \tfrac{\bar{\sigma}_i}{\sigma_i}\right)(1 - b_i^s) \\
\zeta_i^x &= \gamma\bar{\sigma}_i(1 + \bar{\sigma}_p^{-1}\bar{\sigma}_\eta)
\end{aligned}$$

Proof.

$$\begin{aligned}
r^e &= f - \Gamma\tilde{p} \\
&= \gamma\Gamma\bar{\Sigma}\bar{x} + \Gamma(I - A_z)z - \Gamma A_z(I - b^s)B - \Gamma A_x x \\
&= \Gamma\left(\gamma\bar{\Sigma}\bar{x} + \bar{\Sigma}\Sigma^{-1}z - (I - \bar{\Sigma}\Sigma^{-1})(I - b^s)B + \gamma\bar{\Sigma}\left(I + \bar{\Sigma}_p^{-1}\bar{\Sigma}_\eta\right)x\right)
\end{aligned}$$

Given the matrix structure of Γ, each stock's excess return can be written as a part that loads on the market excess return, and a part that contingent on the shocks. □

Corollary A1 shows that the excess return of a stock depends on three part: (i) a constant determined by its idiosyncratic volatility and supply; (ii) a part that depends on the market excess return and its exposure; and (iii) three shocks with stock-specific loadings. Specifically, the model implies that when analysts' bias predicts returns negatively, consistent with the empirical findings. The predictability is weaker for stock with more information acquisition (i.e., higher b) as shown in the following corollary.

Corollary A2. *The analysts' bias B_i negatively predict stock excess return. If χ is sufficiently large, the predictability is decreasing with the de-biasing activity b_i^s.*

Proof. Note that

$$\zeta_i^B = \left(1 - \frac{\bar{\sigma}_i}{\sigma_i}\right)(1 - b_i^s) = \frac{\bar{\sigma}_{\eta i}^{-1} + \bar{\sigma}_{pi}^{-1}}{\sigma^{-1} + \bar{\sigma}_{\eta i}^{-1} + \bar{\sigma}_{pi}^{-1}}(1 - b_i^s) = \frac{1 + \frac{\bar{\sigma}_{\eta i}}{\bar{\sigma}_{pi}}}{\frac{\bar{\sigma}_{\eta i}}{\sigma_i} + 1 + \frac{\bar{\sigma}_{\eta i}}{\bar{\sigma}_{pi}}}(1 - b_i^s)$$

where $\bar{\sigma}_{\eta i} = \frac{1}{\chi}\sigma_i^B(1 - b_i^s)^2$ and $\bar{\sigma}_{pi} = \frac{1}{1-\chi}\left(\chi\bar{\sigma}_{\eta i} + \gamma^2\sigma_{xi}\bar{\sigma}_{\eta i}^2\right)$. In addition, given $\sigma_i^B = \rho\sigma_i$, we obtain

$$\zeta_i^B = \frac{1 + \frac{1-\chi}{\chi + \gamma^2\sigma_{xi}\bar{\sigma}_{\eta i}}}{\frac{\rho}{\chi}(1 - b_i^s)^2 + 1 + \frac{1-\chi}{\chi + \gamma^2\sigma_{xi}\bar{\sigma}_{\eta i}}}(1 - b_i^s) = \frac{1}{\frac{\rho}{\chi}\frac{\chi + \gamma^2\sigma_{xi}\bar{\sigma}_{\eta i}}{1 + \gamma^2\sigma_{xi}\bar{\sigma}_{\eta i}}(1 - b_i^s) + \frac{1}{1 - b_i^s}} \tag{A15}$$

Denote $g(b_i^s) \equiv \frac{\rho}{\chi}\frac{\chi + \gamma^2\sigma_{xi}\bar{\sigma}_{\eta i}}{1 + \gamma^2\sigma_{xi}\bar{\sigma}_{\eta i}}$. We show that the denominator in Equation A15 is increasing in b_i^s when χ is large enough. Note that the derivative of the denominator with respect to b_i^s is given by $-g(b_i^s) + \frac{1}{(1-b_i^s)^2} + \frac{dg(b_i^s)}{db_i^s}(1 - b_i^s)$ where

$$\frac{dg(b_i^s)}{db_i^s} = \frac{\rho(1-\chi)}{\chi}\frac{\gamma^2\sigma_{xi}\frac{d\bar{\sigma}_{\eta i}}{db_i^s}}{(1 + \gamma^2\sigma_{xi}\bar{\sigma}_{\eta i})^2} = -\frac{\rho(1-\chi)}{\chi^2}\frac{2\gamma^2\sigma_{xi}\sigma_i^B(1 - b_i^s)}{(1 + \gamma^2\sigma_{xi}\bar{\sigma}_{\eta i})^2}$$

Now, when χ is sufficiently high (i.e., close to 1), this derivative converges to zero. In addition, $g(b_i^s)$ will converge to $\rho < 1$. Note that $\frac{1}{(1-b_i^s)^2} \geq 1$. Therefore, there exist a χ^* such that when $\chi > \chi^*$, $-g(b_i^s) + \frac{1}{(1-b_i^s)^2} + \frac{dg(b_i^s)}{db_i^s}(1 - b_i^s) > 0$. That is, the denominator in Equation A15 is increasing in b_i^s, and thus the return predictability is decreasing in b_i^s. □

Information decision At $t = 0$, investors choose posterior precision of the the de-biased through information acquisition to maximize time-0 expected utility U_{0j}.

The proof of Lemma A2 shows that the time-0 utility can be written as the following form

$$U_{0j} = constant + \sum_{i=1}^{n}\left(\lambda_i\frac{\tau_{ij}^{\eta}}{\tau_i^B} - \frac{\kappa_i}{2}\left(\frac{\tau_{ij}^{\eta}}{\tau_i^B} - 1\right)^2\right) \tag{A16}$$

where λ_i is the marginal benefit of increasing relative signal precision (de-biasing), which depends on the aggregate posterior variances. Importantly, λ_i does not depend on investor j's decision, since any investor is atomic and cannot affect the aggregate posterior variances. Then the optimization problem is quite straightforward: each skilled investor chooses an optimal level of τ_{ij}^{η} for each stock to maximize her utility in Equation A16. Then we reach the following lemma on optimal information acquisition.

Lemma A2. *An skilled investor j choose the optimal signal precision of de-biased signal as follows*

$$\tau_{ij}^{\eta} = \tau_i^B \left(1 + \frac{\lambda_i}{\kappa_i}\right) \tag{A17}$$

where

$$\lambda_i = \frac{1}{2\gamma\sigma_i^B}\left((1-\chi)\bar{\sigma}_i + \bar{\sigma}_i^2\left[\bar{\sigma}_{\eta i}^{-1} + \chi\sigma_i^{-1} + \gamma^2(\sigma_{xi} + \bar{x}_i^2)\right]\right) \tag{A18}$$

Proof. Put the expression of the demand function $\tilde{q}_j$ to U_{0j},

$$\begin{aligned} U_{0j} &= W_0 + E_0\left[\tilde{q}_j' E_j\left(\tilde{f} - \tilde{p}\right) - \frac{\gamma}{2}\tilde{q}_j' V_j\left(\tilde{f} - \tilde{p}\right)\tilde{q}_j\right] - \sum_{i=1}^{n} c_{ij} \\ &= W_0 + \frac{1}{\gamma}E_0\left[\left(E_j(\tilde{f}) - \tilde{p}\right)'\hat{\Sigma}_j^{-1}\left(E_j(\tilde{f}) - \tilde{p}\right) - \frac{1}{2}\left(E_j(\tilde{f}) - \tilde{p}\right)'\hat{\Sigma}_j^{-1}\hat{\Sigma}_j\hat{\Sigma}_j^{-1}\left(E_j(\tilde{f}) - \tilde{p}\right)\right] - \sum_{i=1}^{n} c_{ij} \\ &= W_0 + \frac{1}{2\gamma}E_0\left[\left(E_j(\tilde{f}) - \tilde{p}\right)'\hat{\Sigma}_j^{-1}\left(E_j(\tilde{f}) - \tilde{p}\right)\right] - \sum_{i=1}^{n} c_{ij} \end{aligned}$$

Note that $E_j(\tilde{f}) - \tilde{p}$ is normally distributed. Thus U_{0j} is an expectation of a non-central χ^2-distributed random variable. According to Van Nieuwerburgh and Veldkamp (2010), this equals

$$\begin{aligned} U_{0j} &= W_0 + \frac{1}{2\gamma}\left[\text{Trace}\left[\hat{\Sigma}_j^{-1}V_0\left(E_j(\tilde{f}) - \tilde{p}\right)\right] + E_0\left(E_j(\tilde{f}) - \tilde{p}\right)'\hat{\Sigma}_j^{-1}E_0\left(E_j(\tilde{f}) - \tilde{p}\right)\right] - \sum_{i=1}^{n} c_{ij} \\ &= W_0 + \frac{1}{2\gamma}\left[\text{Trace}\left[\hat{\Sigma}_j^{-1}V_0\left(\tilde{f} - \tilde{p}\right) - I\right] + \gamma^2\bar{x}'\bar{\Sigma}'\hat{\Sigma}_j^{-1}\bar{\Sigma}\bar{x}\right] - \sum_{i=1}^{n} c_{ij} \end{aligned}$$

where Trace$(\cdot)$ is the trace of a matrix. The second equality applies the Law of Total Variance,

$$V_0\left(E_j\left(\tilde{f}\right) - \tilde{p}\right) = V_0\left(\tilde{f} - \tilde{p}\right) - E_0\left(V_j\left(\tilde{f}_j - \tilde{p}\right)\right) = V_0\left(\tilde{f} - \tilde{p}\right) - \hat{\Sigma}_j$$

Note that

$$\begin{aligned}
V &\equiv V_0\left(\tilde{f}-\tilde{p}\right)\\
&= (I-A_z)\Sigma(I-A_z)' + A_B\Sigma_B A_B' + A_x\Sigma_x A_x'\\
&= (I-A_z)\Sigma(I-A_z)' + A_z\Sigma_p A_z'\\
&= \bar{\Sigma}\left[\Sigma^{-1} + (\bar{\Sigma}_p^{-1}+\bar{\Sigma}_\eta^{-1})\Sigma_p(\bar{\Sigma}_p^{-1}+\bar{\Sigma}_\eta^{-1})'\right]\bar{\Sigma}'\\
&= \bar{\Sigma}\left[\Sigma^{-1} + (1-\chi)\bar{\Sigma}_p^{-1} + 2(1-\chi)\bar{\Sigma}_\eta^{-1} + \bar{\Sigma}_\eta^{-1}\Sigma_p\bar{\Sigma}_\eta^{\prime -1}\right]\bar{\Sigma}'\\
&= \bar{\Sigma}\left[\chi\Sigma^{-1} + (1-\chi)\bar{\Sigma}^{-1} + (1-\chi)\bar{\Sigma}_\eta^{\prime -1} + \chi\bar{\Sigma}_\eta^{\prime -1} + \gamma^2\Sigma_x\right]\bar{\Sigma}'\\
&= \bar{\Sigma}\left[\chi\Sigma^{-1} + (1-\chi)\bar{\Sigma}^{-1} + \bar{\Sigma}_\eta^{\prime -1} + \gamma^2\Sigma_x\right]\bar{\Sigma}'
\end{aligned}$$

where A_z, A_B, and A_x are given in Lemma A1. The ith diagonal element of V is then

$$V_{ii} = (1-\chi)\bar{\sigma}_i + \bar{\sigma}_i^2\left[\bar{\sigma}_{\eta i}^{-1} + \chi\sigma_i^{-1} + \gamma^2\sigma_{xi}\right]$$

Given the diagonal nature of the problem, the ex-ante expected utility is given by

$$\begin{aligned}
U_{0j} =& W_0 + \frac{1}{2\gamma}\sum_{i=1}^n\left(\sigma_i^{-1}+\tau_{ij}^\eta+\bar{\sigma}_{pi}^{-1}\right)V_{ii} - \frac{n}{2\gamma} + \frac{1}{2\gamma}\sum_{i=1}^n\gamma^2\bar{\sigma}_i^2\bar{x}_i^2\left(\sigma_i^{-1}+\tau_{ij}^\eta+\bar{\sigma}_{pi}^{-1}\right) - \sum_{i=1}^n c_{ij}\\
=& constant + \frac{1}{2\gamma}\sum_{i=1}^n\tau_{ij}^\eta\left(V_{ii}+\gamma^2\bar{\sigma}_i^2\bar{x}_i^2\right) - \sum_{i=1}^n c_{ij}\\
=& constant + \sum_{i=1}^n\lambda_i\frac{\tau_{ij}^\eta}{\tau_i^B} - \sum_{i=1}^n\frac{\kappa_i}{2}\left(\frac{\tau_{ij}^\eta}{\tau_i^B}-1\right)^2
\end{aligned}$$

where $\lambda_i = \frac{1}{2\gamma\sigma_i^B}\left(V_{ii}+\gamma^2\bar{\sigma}_i^2\bar{x}_i^2\right)$.

The time-0 expected utility is a *linear function* on the precision of the de-biased signal τ_{ij}^η with a quadratic information cost. Therefore the optimal learning decision is given by $\tau_{ij}^\eta = \tau_i^B(1+\frac{\lambda_i}{\kappa_i})$. □

The equilibrium is such that skilled investors choose the same de-baised signal precision τ_{ij}^η following Equation A17 where λ_i satisfies Equation A18, which is determined by investors' aggregated signal precision. Denote $\theta_i = \frac{\tau_{ij}^\eta}{\tau_i^B} = \frac{\bar{\tau}_{\eta i}}{\chi\tau_i^B}$ as skilled investors' relative signal precision, the equilibrium is characterized by the fixed-point problem below

$$f(\theta_i) \equiv \kappa_i\left(\theta_i - 1\right) - \lambda_i = 0 \tag{A19}$$

C.2.3 Implications

Corollary A3. *The marginal benefit of increasing relative signal precision, λ_i, is decreasing in the de-biasing level b_i^s.*

Proof. Note that the aggregated signal precision is increasing in the de-biasing level $\bar{\tau}_{\eta i} = \frac{\chi}{\sigma_i^B(1-b_i^s)^2}$. Therefore, to show that $\frac{d\lambda_i}{db_i^s} > 0$ we show the equivalent statement $\frac{d\lambda_i}{d\bar{\tau}_{\eta i}} > 0$ Note that λ_i can be writen as

$$\lambda_i = \frac{1}{2\gamma\sigma_i^B}\frac{(1-\chi)\bar{\tau}_i + \gamma^2(\sigma_{xi}+\bar{x}_i^2) + \bar{\tau}_{\eta i} + \chi\tau_i}{\bar{\tau}_i^2}$$

where $\bar{\tau}_i = \bar{\sigma}_i^{-1}$ and $\bar{\tau}_{\eta i} = \bar{\sigma}_{\eta i}^{-1}$. Then it can be shown that

$$\begin{aligned}
\frac{d\lambda_i}{d\bar{\tau}_{\eta i}} &= \frac{1}{2\gamma\sigma_i^B}\frac{\left((1-\chi)\frac{d\bar{\tau}_i}{d\bar{\tau}_{\eta i}}+1\right)\bar{\tau}_i^2 - 2\bar{\tau}_i\frac{d\bar{\tau}_i}{d\bar{\tau}_{\eta i}}\left((1-\chi)\bar{\tau}_i + \gamma^2(\sigma_{xi}+\bar{x}_i^2)+\bar{\tau}_{\eta i}+\chi\tau_i\right)}{\bar{\tau}_i^4} \\
&= \frac{1}{2\gamma\sigma_i^B}\frac{\left((1-\chi)\frac{d\bar{\tau}_i}{d\bar{\tau}_{\eta i}}+1\right)\bar{\tau}_i - 2\frac{d\bar{\tau}_i}{d\bar{\tau}_{\eta i}}\left(\bar{\tau}_i + \gamma^2(\sigma_{xi}+\bar{x}_i^2)+(1-\chi)\bar{\tau}_{\eta i}-\chi\bar{\tau}_{pi}\right)}{\bar{\tau}_i^3} \\
&= \frac{1}{2\gamma\sigma_i^B}\frac{-\left((1+\chi)\frac{d\bar{\tau}_i}{d\bar{\tau}_{\eta i}}-1\right)\bar{\tau}_i - 2\frac{d\bar{\tau}_i}{d\bar{\tau}_{\eta i}}\gamma^2(\sigma_{xi}+\bar{x}_i^2) - 2\frac{d\bar{\tau}_i}{d\bar{\tau}_{\eta i}}\left((1-\chi)\bar{\tau}_{\eta i}-\chi\bar{\tau}_{pi}\right)}{\bar{\tau}_i^3}
\end{aligned}$$

The numerator above is negative because (i) $\frac{d\bar{\tau}_i}{d\bar{\tau}_{\eta i}} > 1$, and (ii) $(1-\chi)\bar{\tau}_{\eta i} > \chi\bar{\tau}_{pi}$. We prove below.

(i) Using the relation $\bar{\sigma}_{pi} = \frac{1}{1-\chi}\left(\chi\bar{\sigma}_{\eta i} + \gamma^2\sigma_{xi}\bar{\sigma}_{\eta i}^2\right)$, we can get

$$\frac{d\bar{\sigma}_{pi}^{-1}}{d\bar{\sigma}_{\eta i}^{-1}} = \frac{d\bar{\sigma}_{pi}}{d\bar{\sigma}_{\eta i}}\frac{\bar{\sigma}_{\eta i}^2}{\bar{\sigma}_{pi}^2} = (1-\chi)\frac{\chi + 2\gamma^2\sigma_{xi}\bar{\sigma}_{\eta i}}{(\chi+\gamma^2\sigma_{xi}\bar{\sigma}_{\eta i})^2} > 0 \tag{A20}$$

Thus $\frac{d\bar{\tau}_i}{d\bar{\tau}_{\eta i}} = 1 + \frac{d\bar{\tau}_{pi}}{d\bar{\tau}_{\eta i}} > 1$.

(ii) Given that

$$(1-\chi)\bar{\tau}_{\eta i} - \chi\bar{\tau}_{pi} = (1-\chi)\chi\left(\sigma_{\eta i}^{-1} - \sigma_{pi}^{-1}\right) > 0$$

The inequality holds because the price signal is less precise than the private signal, that is, $\frac{1}{\sigma_{\eta i}} > \frac{1}{\sigma_{pi}} = \frac{1}{\sigma_{\eta i}+\gamma^2\sigma_{xi}\bar{\sigma}_{\eta i}^2}$.

□

Proposition A1. *If χ is sufficiently large, a higher intrinsic uncertainty σ_i^F lowers de-biasing activity (less information acquisition) and increases return predictability of analysts' forecast biases*

Proof. We first prove that the de-biasing level is decreasing in the intrinsic volatility, i.e., $\frac{db_i}{d\sigma_i^F} < 0$. This is equivalent to show that the relative signal precision θ_i is decreasing in σ_i^F, since there is a positive monotonic relationship between θ_i and b_i given by $\theta_i = \frac{1}{(1-b_i)^2}$. Note that the equilibrium is determined by solving the fixed-point problem:

$$f(\theta_i) = \kappa_i(\theta_i - 1) - \lambda_i = 0$$

Applying the Implicit Function Theorem,

$$\frac{d\theta_i}{d\sigma_i^F} = -\frac{\partial f/\partial\sigma_i^F}{\partial f/\partial\theta_i} = -\frac{\psi(\theta_i - 1) - \frac{\partial\lambda_i}{\partial\sigma_i^F}}{\kappa_i - \frac{\partial\lambda_i}{\partial\theta_i}} = \frac{\frac{\partial\lambda_i}{\partial\sigma_i^F} - \frac{\lambda_i}{\sigma_i^F}}{\kappa_i - \frac{\partial\lambda_i}{\partial\theta_i}} \tag{A21}$$

According to the proof of Corollary A3, if χ is sufficiently large, $\frac{\partial\lambda_i}{\partial b_i^s} < 0$ and thus $\frac{\partial\lambda_i}{\partial\theta_i} < 0$. Therefore the denominator in Equation A21 is positive.

We will now prove that if χ is large enough, the numerator in Equation A21 is negative. Note that λ_i can be written as

$$\lambda = \frac{1}{2\gamma}\left((1-\chi)\frac{\bar{\sigma}_i}{\sigma_i^B} + \left(\frac{\bar{\sigma}_i}{\sigma_i^B}\right)^2\left[\frac{\sigma_i^B}{\bar{\sigma}_{\eta i}} + \chi\frac{\sigma_i^B}{\sigma_i} + \gamma^2(\sigma_{xi} + \bar{x}_i^2)\rho\sigma_i\right]\right)$$

Also note that $\sigma_i^B = \rho\sigma_i$ and $\bar{\sigma}_{\eta i} = \frac{1}{\chi}\sigma_i^B(1-b_i^s)^2$. Thus $Q_i \equiv \frac{\sigma_i^B}{\bar{\sigma}_{\eta i}} + \chi\frac{\sigma_i^B}{\sigma_i}$ does not depend on σ. Denote

$$P_i \equiv \frac{\bar{\sigma}_i}{\sigma_i^B} = \frac{\sigma_i^{B^{-1}}}{\sigma_i^{-1} + \bar{\sigma}_{\eta i}^{-1} + \bar{\sigma}_{pi}^{-1}} = \frac{1}{\rho + \chi(1-b_i^s)^{-2} + \frac{1-\chi}{(1-b_i^s)^2 + \gamma^2\sigma_{xi}\rho\sigma_i(1-b_i^s)^4/\chi^2}}$$

and $C_i \equiv \rho\gamma^2(\sigma_{xi} + \bar{x}_i^2)$ Then λ_i can be written as

$$\lambda_i = \frac{1}{2\gamma}\left((1-\chi)P_i + P_i^2(Q_i + C_i\sigma_i)\right)$$

Note that $\sigma_i = \sigma_i^F + \sigma_i^S$, thus

$$\frac{\partial\lambda_i}{\partial\sigma_i^F} = \frac{\partial\lambda_i}{\partial\sigma_i} = \frac{1}{2\gamma}\left((1-\chi)\frac{\partial P_i}{\partial\sigma_i} + 2P_i\frac{\partial P_i}{\partial\sigma_i}(Q_i + C_i\sigma_i) + P_i^2 C_i\right)$$

where

$$\frac{\partial P_i}{\partial\sigma_i} = P_i^2\frac{(1-\chi)\gamma^2\sigma_{xi}\rho(1-b_i^s)^4/\chi^2}{((1-b_i^s)^2 + \gamma^2\sigma_{xi}\rho\sigma_i(1-b_i^s)^4/\chi^2)^2}$$

Note that the numerator in Equation A21 is

$$\frac{\partial \lambda_i}{\partial \sigma_i^F} - \frac{\lambda_i}{\sigma_i^F} = \frac{1}{2\gamma}\left((1-\chi)\left(\frac{\partial P_i}{\partial \sigma_i} - \frac{P_i}{\sigma_i^F}\right) + 2P_i\frac{\partial P_i}{\partial \sigma_i}(Q_i + C_i\sigma_i) - \frac{P_i^2 Q_i}{\sigma_i^F}\right) \tag{A22}$$

If χ is large enough, then $\frac{\partial P_i}{\partial \sigma_i}$ will be close to zero. Thus there exist a cutoff χ^*, when $\chi > \chi^*$, Equation A22 is negative. Given that the denominator is positive, this means that $\frac{d\theta_i}{d\sigma_i^F} < 0$ and $\frac{db_i^s}{d\sigma_i^F} < 0$.

Next, we prove that $\frac{d\zeta_i^B}{d\sigma_i^F} > 0$. Note that

$$\frac{d\zeta_i^B}{d\sigma_i^F} = \frac{\partial \zeta_i^B}{\partial b_i^s}\frac{db_i^s}{d\sigma_i^F} + \frac{\partial \zeta_i^B}{\partial \sigma_i^F}$$

According to Corollary A2, $\frac{\partial \zeta_i^B}{\partial b_i^s} < 0$, and we have proved $\frac{db_i^s}{d\sigma_i^F} < 0$. To derive $\frac{\partial \zeta_i^B}{\partial \sigma_i^F}$, first note that

$$\begin{aligned}\zeta_i^B &= \left(1 - \frac{\bar{\sigma}_i}{\sigma_i}\right)(1-b_i^s) = \left(1 - \frac{\sigma_i^{-1}}{\sigma_i^{-1} + \bar{\sigma}_{\eta i}^{-1} + \bar{\sigma}_{pi}^{-1}}\right)(1-b_i^s) \\ &= \left(1 - \frac{\rho}{\rho + \chi(1-b_i^s)^{-2} + \frac{(1-\chi)\chi^2}{(1-b_i^s)^2 + \gamma^2\sigma_{xi}\rho\sigma_i(1-b_i^s)^4}}\right)(1-b_i^s)\end{aligned}$$

Therefore, all else equal, ζ_i^B is decreasing in σ_i, thus $\frac{\partial \zeta_i^B}{\partial \sigma_i^F} = \frac{\partial \zeta_i^B}{\partial \sigma_i} < 0$. However, if χ is sufficiently large, this partial derivative is converging to zero. Therefore, if χ is large enough, $\frac{d\zeta_i^B}{d\sigma_i^F} > 0$: the return predictability is increasing in the intrinsic uncertainty σ_i^F.

□

Proposition A2. *If χ is sufficiently large, a higher temporal uncertainty σ_i^S increases debiasing activity (more information acquisition) and decreases return predictability of analysts' forecast biases*

Proof. The proof follows the same strategy as that of Proposition A1. We first prove that $\frac{d\theta_i}{d\sigma_i^S} > 0$. Applying the Implicit Function Theorem,

$$\frac{d\theta_i}{d\sigma_i^S} = -\frac{\partial f/\partial \sigma_i^S}{\partial f/\partial \theta_i} = \frac{\frac{\partial \lambda_i}{\partial \sigma_i^S}}{\kappa_i - \frac{\partial \lambda_i}{\partial \theta_i}} \tag{A23}$$

We have shown in the proof of Proposition A1 that the denominator in Equation A23 is positive. In addition,

$$\frac{\partial \lambda_i}{\partial \sigma_i^S} = \frac{\partial \lambda_i}{\partial \sigma_i} = \frac{1}{2\gamma}\left((1-\chi)\frac{\partial P_i}{\partial \sigma_i} + 2P_i\frac{\partial P_i}{\partial \sigma_i}(Q_i + C_i\sigma_i) + P_i^2 C_i\right)$$

Note that $\frac{\partial P_i}{\partial \sigma_i} > 0$ and $P_i^2 C_i > 0$. Therefore, $\frac{\partial \lambda_i}{\partial \sigma_i^S} > 0$ and thus $\frac{d\theta_i}{d\sigma_i^S} > 0$ and $\frac{db_i^s}{d\sigma_i^S} > 0$.

Next, we prove that $\frac{d\zeta_i^B}{d\sigma_i^S} < 0$. Note that

$$\frac{d\zeta_i^B}{d\sigma_i^S} = \frac{\partial \zeta_i^B}{\partial b_i^s}\frac{db_i^s}{d\sigma_i^S} + \frac{\partial \zeta_i^B}{\partial \sigma_i^S}$$

According to Corollary A3, if χ is sufficiently large, $\frac{\partial \zeta_i^B}{\partial b_i^s} < 0$, and we have proved $\frac{db_i^s}{d\sigma_i^S} > 0$ above. In addition, in the proof of Proposition A1, we show that $\frac{\partial \zeta_i^B}{\partial \sigma_i} < 0$ and thus $\frac{\partial \zeta_i^B}{\partial \sigma_i^S} < 0$. Therefore, if χ is large enough, $\frac{d\zeta_i^B}{d\sigma_i^S} < 0$: the return predictability is increasing in the intrinsic uncertainty σ_i^S.

□

Proposition A3. *If χ is sufficiently large, a higher intrinsic (temporal) uncertainty decreases (increases) the price efficiency, as measured by the price response to the fundamental shock, $A_{z,i}$, and increases (decreases) the fundamental-based return anomaly, as measured by the return predictability of the fundamental shock, ζ_i^z.*

Proof. We first prove for σ_i^F. Note that

$$\frac{dA_{z,i}}{d\sigma_i^F} = \frac{\partial A_{z,i}}{\partial b_i^s}\frac{db_i^s}{d\sigma_i^F} + \frac{\partial A_{z,i}}{\partial \sigma_i^F}$$

Note that

$$A_{z,i} = 1 - \frac{\sigma_i^{-1}}{\bar{\sigma}_i^{-1}} = 1 - \frac{\rho}{\rho + \chi(1-b_i^s)^{-2} + \frac{(1-\chi)\chi^2}{(1-b_i^s)^2 + \gamma^2\sigma_{xi}\rho\sigma_i(1-b_i^s)^4}}$$

Thus $\frac{\partial A_{z,i}}{\partial b_i^s} > 0$ and $\frac{\partial A_{z,i}}{\partial \sigma_i^F} < 0$. In addition, $\frac{db_i^s}{d\sigma_i^F} < 0$ according to Proposition A1. Thus $\frac{dA_{z,i}}{d\sigma_i^F} < 0$.

For σ_i^S,

$$\frac{dA_{z,i}}{d\sigma_i^S} = \frac{\partial A_{z,i}}{\partial b_i^s}\frac{db_i^s}{d\sigma_i^S} + \frac{\partial A_{z,i}}{\partial \sigma_i^S}$$

where $\frac{\partial A_{z,i}}{\partial b_i^s} > 0$, $\frac{db_i^s}{d\sigma_i^S} > 0$, and $\frac{\partial A_{z,i}}{\partial \sigma_i^S} < 0$. But note that when χ is sufficiently large, $\frac{\partial A_{z,i}}{\partial \sigma_i^S} \to 0$. Therefore, there exist a χ^*, when $\chi > \chi^*$, $\frac{dA_{z,i}}{d\sigma_i^S} > 0$.

Note that $\zeta_i^z = 1 - A_{z,i}$, the opposite of the above predictions holds for the fundamental-based return anomaly.

$\square$